MA/PHY506 Fall 2015 Problem Set 9 DUE: 20 November 2015

1. Let ϕ_j be an orthonormal basis of the Hilbert space $L^2([a, b])$. For any $f \in L^2([a, b])$, the mean square (MS) error between f and the finite series approximation $S_N(x) = \sum_{j=1}^N c_j \phi_j(x)$ is defined by

$$MS_N(f; \{c_j\}) \equiv \int_a^b |f(x) - S_N(x)|^2 dx.$$

Assume that f, the coefficients c_j , and the basis functions ϕ_j are all real (for simplicity). Show that $MS(f; \{c_j\})$ is minimized with the choice $c_j = \int_a^b \phi_j(x) f(x) dx$, the expansion coefficients of f relative to the orthonormal basis ϕ_j .

- 2. Consider the nonhomogeneous BVP: $y'' = x(x-2\pi)$ on $[0,\pi]$. Expand y in the eigenfunctions of the related Sturm-Liouville problem $Ly = -y'' = \lambda y$ with DBC at 0 and π . Expand $h(x) = x(x-2\pi)$ in the eigenfunctions of this Sturm-Liouville problem. Find a formal series solution for y.
- 3. Find the Fourier series for a square wave:

$$f(x) = \begin{cases} h/2 & 0 < x < \pi \\ -h/2 & -\pi < x < 0 \end{cases}$$

What is the value of the series at $x = -\pi, 0, \pi$? Is this reasonable?