1. Let \( \phi_j \) be an orthonormal basis of the Hilbert space \( L^2([a,b]) \). For any \( f \in L^2([a,b]) \), the mean square (MS) error between \( f \) and the finite series approximation \( S_N(x) = \sum_{j=1}^{N} c_j \phi_j(x) \) is defined by

\[
MS_N(f; \{c_j\}) \equiv \int_a^b |f(x) - S_N(x)|^2 \, dx.
\]

Assume that \( f \), the coefficients \( c_j \), and the basis functions \( \phi_j \) are all real (for simplicity). Show that \( MS(f; \{c_j\}) \) is minimized with the choice \( c_j = \int_a^b \phi_j(x) f(x) \, dx \), the expansion coefficients of \( f \) relative to the orthonormal basis \( \phi_j \).

2. Consider the nonhomogeneous BVP: \( y'' = x(x-2\pi) \) on \([0,\pi]\). Expand \( y \) in the eigenfunctions of the related Sturm-Liouville problem \( Ly = -y'' = \lambda y \) with DBC at 0 and \( \pi \). Expand \( h(x) = x(x-2\pi) \) in the eigenfunctions of this Sturm-Liouville problem. Find a formal series solution for \( y \).

3. Find the Fourier series for a square wave:

\[
f(x) = \begin{cases} 
  \frac{h}{2} & 0 < x < \pi \\
  -\frac{h}{2} & -\pi < x < 0 
\end{cases}
\]

What is the value of the series at \( x = -\pi, 0, \pi \)? Is this reasonable?