MA/PHY506 Fall 2017
Problem Set 9
DUE: 8 December 2017

1. Arfken, Chapter 8, pages 387, problem 8.2.1.

2. Arfken, Chapter 8, page 394: problem 8.3.2.

3. Show that the linear operator $L = -d^2/dx^2$ on $L^2([0, 2\pi])$ is hermitian on the functions that satisfy periodic boundary conditions: $y(0) = y(2\pi)$ and $y'(0) = y'(2\pi)$, and that are twice differentiable. That is, for any two such functions

$$\int_0^{2\pi} \overline{f}(x)(Lg)(x)\ dx = \int_0^{2\pi} \overline{Lf}(x)g(x)\ dx.$$

   Find the normalized eigenfunctions of L, that is, functions satisfying $Lf = \lambda f$, with these properties, and the corresponding eigenfunctions. Check that the eigenfunctions are orthogonal.

4. Consider the nonhomogeneous BVP: $y'' = x(x-2\pi)$ on $[0, \pi]$. Expand y in the eigenfunctions of the related Sturm-Liouville problem $Ly = -y'' = \lambda y$ with DBC at 0 and π. Expand $h(x) = x(x - 2\pi)$ in the eigenfunctions of this Sturm-Liouville problem. Find a formal series solution for y.

5. Find the Fourier series for a square wave:

$$f(x) = \begin{cases} h/2 & 0 < x < \pi \\ -h/2 & -\pi < x < 0 \end{cases}$$

   What is the value of the series at $x = -\pi, 0, \pi$? Is this reasonable?