PHY/MA 506

PS 1-1

# Solutions to PS #1

1

$$(s^2+1)f'(s)+sf(s)=0$$

Separate variables $f$ & $s$: $\frac{f'}{f} = \frac{-s}{s^2+1} = \frac{d}{ds}(\log f(s))$

Integrate: $\log f(s) = -\frac{1}{2}\int \frac{2s\,ds}{s^2+1} = -\frac{1}{2}\log(s^2+1)+C$

$$f(s) = Ae^{-\frac{1}{2}\log(s^2+1)} = \frac{A}{\sqrt{s^2+1}}$$

Check: $f'(s) = \left(-\frac{1}{2}\right)(s^2+1)^{-3/2}\, 2sA$

$= -s(1+s^2)^{-1}f(s)$ ✓

2. $ty'(t)+2y(t) = t^2-t \Rightarrow y'(t)+\frac{2}{t}y(t) = t-1$

Integrating factor $p(s)=\frac{2}{s}$, $\mu(t) = e^{\int^t p(s)ds} = t^2$

$$\frac{d}{dt}[\mu(t)y(t)] = \mu(t)g(t) = t^2[t-1] = t^3-t^2$$

$$t^2y(t) = \frac{1}{4}t^4 - \frac{1}{3}t^3 + C$$

$$\boxed{y(t) = \frac{1}{4}t^2 - \frac{1}{3}t + C/t^2}$$

Check: $y'(t) = \frac{1}{2}t - \frac{1}{3} - \frac{2C}{t^3}$ $\quad ty'(t) = \frac{1}{2}t^2 - \frac{2C}{t^2} - \frac{1}{3}t$

$$2y(t) = \frac{1}{2}t^2 - \frac{2}{3}t + \frac{2C}{t^2}$$

so:

$$ty'+2y = t^2-t$$ ✓

PS1-2

3. $2x + y + xy' = 0$ or $xy' + y = -2x$

$$y' + \frac{1}{x}y = -2$$

$$M(x) = x = e^{\int^x \frac{1}{s}ds}$$

$$\frac{d}{dx}[xy(x)] = -2x \Rightarrow xy(x) = -x^2 + C$$

$$\boxed{y(x) = -x + \frac{C}{x}}$$

Check:

$$y'(x) = -1 - \frac{C}{x^2} \qquad xy'(x) = -x - \frac{C}{x}$$

$$xy' + y = -2x \checkmark$$

4. $y'' + 2y' + 2y = 0 \qquad r^2 + 2r + 2 = 0$

roots: $b^2 - 4ac = -4 < 0$ complex conjugate pair of roots.

$$r_1 = -1 + i \quad r_2 = \bar{r}_1 = -1 - i$$

$$e^{r_1 x} = e^{-x}(\cos x + i \sin x)$$

2 real solns. $y_1(x) = e^{-x}\cos x$, $y_2(x) = e^{-x}\sin x$

These are L.I.

General real soln:

$$y(x) = Ae^{-x}\cos x + Be^{-x}\sin x$$

PS1-3

5. $y'' + 5y' + 6y = 0$

$r^2 + 5r + 6 = 0 = (r+2)(r+3)$

$b^2 - 4ac = 25 - 24 = 1$

$r_{1,2} = \frac{-5 \pm 1}{2} = -3, -2.$

2 independent fncs. $\begin{cases} y_1(x) = e^{-2x} \\ y_2(x) = e^{-3x} \end{cases}$

Basis of soln. space so general soln is:

$$y(x) = Ae^{-2x} + Be^{-3x}$$