**MA/PHY506 Fall 2018**
**Problem Set 9**
**DUE: 3 December 2018**

1. Show that the delta function satisfies the following identity. Let $g$ be a differentiable function with exactly one zero in $[a, b]$ at $x_0$ and so that $g'(x_0) \neq 0$. Then for any nice test function f:

$$\int_a^b f(x)\ \delta[g(x)]\ dx = \frac{f(x_0)}{|g'(x_0)|}.$$

First assume $g(a) < g(b)$ and then show the other case follows by changing the order of the endpoints.

2. Compute the Fourier transform of the function

$$f(x) = \left\{ \begin{array}{cc} 0 & x < 0 \\ Ae^{-\alpha x} & x \geq 0 \end{array} \right.$$

for nonzero constants $A$ and $\alpha > 0$.

3. Derivative of a delta function. For a test function $f$ in one dimension, formally show that

$$\int_{-\infty}^{\infty} \delta'(x - x_0) f(x)\ dx = -f'(x_0).$$

4. Compute the Fourier transform of the $1s$ state of the hydrogen atom:

$$\psi(x) = C_0 e^{-\alpha \|x\|},$$

for $x \in R^3$ and where $C_0$ and $\alpha > 0$ are positive constants. It is convenient to use the integration method:

$$\int e^{-\alpha r}\ r^2 dr = \frac{d^2}{d\alpha^2} \int e^{-\alpha r}\ dr,$$

and problem 2.