

**MA/PHY507 Spring 2016**  
**Problem Set 3**  
**DUE: Friday, 12 February 2016**

1. By using a simple change of variables from  $(x, y)$  to  $(R, \theta)$ , where  $r^2 = x^2 + y^2$  and  $\theta = \tan^{-1}(y/x)$ , show that the Cauchy-Riemann equations in polar coordinates are:

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta} \quad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}.$$

2. Do the following contour integrals using the simplest method you can think of and state if either integral is path dependent:

(a)

$$\int_{\gamma} z^2 dz, \quad \gamma(t) = e^{it} \sin^3 t, \quad 0 \leq t \leq \pi/2.$$

(b)

$$\int_{\gamma} y dz,$$

where  $\gamma$  joins 0 to  $i$  and then  $i$  to  $i + 2$ .

3. Arfken, section 11.2, page 477, problem 11.2.10 a).  
4. Arfken, section 11.3, page 485, problems 11.3.4, 11.3.6, and 11.3.7.