PS1-1

PHY/MA 507 Solns. to PS 1.

1.8.4 Use the sum: $\sum_{k=0}^{N} r^k = \frac{1-r^{N+1}}{1-r}$ $(r \neq 1)$

Why? Let $S_N = \sum_{k=0}^{N} r^k$ so $rS_N = S_{N+1} - 1$

and $S_{N+1} = S_N + r^{N+1}$. Solve for S_N.

Apply this to $\sum_{n=0}^{N-1} e^{inx} = \frac{1-e^{iNx}}{1-e^{ix}}$ $(r = e^{ix})$

$$= \frac{e^{iNx/2}}{e^{ix/2}} \frac{(e^{-iNx/2} - e^{iNx/2})}{(e^{-ix/2} - e^{ix/2})}$$

$$= e^{i(N-1)x/2} \frac{\sin\left(\frac{Nx}{2}\right)}{\sin\left(\frac{x}{2}\right)}$$

Take Re & Im parts:

a) $\sum_{n=0}^{N-1} \cos nx = \mathrm{Re}\left(\sum_{n=0}^{\infty} e^{inx}\right) = \cos\left(\frac{(N-1)x}{2}\right) \frac{\sin\left(\frac{Nx}{2}\right)}{\sin\left(\frac{x}{2}\right)}$

b) $\sum_{n=0}^{N-1} \sin nx = \mathrm{Im}\left(\sum_{n=0}^{\infty} e^{inx}\right) = \sin\left(\frac{(N-1)x}{2}\right) \frac{\sin\left(\frac{Nx}{2}\right)}{\sin\left(\frac{x}{2}\right)}$

1.8.5 $i \sin z = \frac{1}{2}(e^{iz} - e^{-iz}) = \sinh(iz)$

$\sin(iz) = \frac{1}{2i}\left[e^{i(iz)} - e^{-i(iz)}\right] = \frac{1}{2i}(e^{-z} - e^{z})$

$= \left(\frac{-1}{i}\right) \cdot \frac{1}{2}(e^{z} - e^{-z}) = i \sinh z$

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$$\cosh iz = \frac{1}{2}\left(e^{iz} + e^{-iz}\right) = \cos z$$

$$\cos(iz) = \frac{1}{2}\left(e^{i(iz)} + e^{-i(iz)}\right) = \frac{1}{2}\left(e^{-z} + e^{z}\right)$$

$$= \cosh z.$$

1.8.6 a) $\underbrace{\sin(x+iy)}_{\sin z} = \frac{1}{2i}\left(e^{i(x+iy)} - e^{-(x+iy)i}\right)$

$$= \frac{1}{2i}\left(e^{ix}e^{-y} - e^{-ix}e^{y}\right)$$

$$= \frac{1}{2i}\left[e^{-y}(\cos x + i\sin x) - e^{y}(\cos x - i\sin x)\right]$$

$$= \frac{1}{2i}\left(e^{-y} - e^{y}\right)\cos x + \frac{1}{2}\sin x\left(e^{-y} + e^{y}\right)$$

$$= i\sinh y\cos x + \cosh y\sin x$$

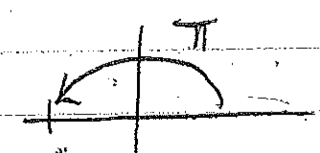
$$= \sin x\cosh y + i\cos x\sinh y$$

b) $|\sin z|^2 = (\cos x\sinh y)^2 + (\sin x\cosh y)^2$

use part(a) $= \sinh^2 y + \sin^2 x\left(\cosh^2 y - \sinh^2 y\right)$

$$= \sinh^2 y + \sin^2 x$$

2. 1.8.10 a) $(-8)^{\frac{1}{3}} = 2e^{i(\pi/3 + 2\pi k/3)}$



$$= 2e^{i\pi/3},\ 2e^{i(\pi)},\ 2e^{i(5\pi/3)}$$

PS 1-3

so the 3 roots are

$r_1 = 2\left(\frac{1}{2} + i\sqrt{3}/2\right) = 1 + i\sqrt{3}$

$r_2 = 2(-1) = -2$

$r_3 = 2\left(\cos 5\pi/3 + i \sin 5\pi/3\right) = 1 - i\sqrt{3}$.

b.) $i^{1/4}$ $\quad |i| = 1 \quad \arg i = \pi/2 \quad$ PB.

$$\frac{\arg z}{N} + \frac{2\pi k}{N} = \frac{\pi}{8} + \frac{\pi k}{2} \quad k = 0, 1, 2, 3$$

$\arg r_1 = \frac{\pi}{8} \qquad \arg r_3 = \frac{9\pi}{8}$

$\arg r_2 = \frac{5\pi}{8} \qquad \arg r_4 = \frac{13\pi}{8}$

roots $\quad r_1 = |i|^{\frac{1}{2}}\left(\cos \pi/8 + i \sin \pi/8\right) = e^{i\pi/8}$

$r_2 = e^{5\pi i/8}$

$r_3 = e^{9\pi i/8}$

$r_4 = e^{13\pi i/8}$

c.) $e^{i\pi/4} = \cos \pi/4 + i \sin \pi/4 = \frac{\sqrt{2}}{2}(1+i)$

1.8.11 a) $(1+i)^3 = \left(\sqrt{2}e^{i\pi/4}\right)^3 = 2^{3/2} e^{3i\pi/4}$

$= 2^{3/2}\left(\cos \frac{3\pi}{4} + i \sin 3\pi/4\right)$

$= 2(-1+i)$

b) $(-1)^{\frac{1}{5}} = e^{i(\pi/5 + 2\pi k/5)} \quad k = 0, 1, 2, 3, 4$

$r_1 = e^{i\pi/5}, \; r_2 = e^{i3\pi/5}, \; r_3 = e^{i\pi}, \; r_4 = e^{7i\pi/5}, \; r_5 = e^{i9\pi/5}$