1 Complex algebra

1. Express $3 + 4i$ in polar form

2. Compute $\Re\{(2 - i)(-1 + 4i)\}$

3. Compute $\Im\left(\frac{1}{3 - 6i}\right)$

4. Find the 5 roots of $z = 2i$

2 Analytic functions

1. Compute the logarithm of $z = 1 + i$ using the principle branch, the branch $[\frac{\pi}{2}, \frac{5\pi}{2})$, and $[-3\pi, -\pi)$.

2. Verify that the function $f(z) = z \cos z$ is entire.

3. Where is the function $g(z) = \log_{\text{PB}}(z + 2)$ analytic?

4. Is the function $u(x, y) = \sin x \cosh y$ harmonic? If yes, find a harmonic conjugate v so that $f = u + iv$ is entire.

3 Contour integrals

1. Compute the contour integral:
$$\int_0^{1+i\pi} e^{2z} \, dz.$$

2. Compute the contour integral:
$$\oint_{|z-i|=1} \frac{e^{z^2}}{(z + 1)(z^2 + 1)} \, dz.$$

4 Taylor and Laurent expansions

1. Find the Laurent expansions of
$$f(z) = \frac{z}{1+z}$$
about $z_0 = 0$ valid for $0 < |z| < 1$ and for $|z| > 1$.

2. Compute $Res(f, 1)$ for
$$f(z) = \frac{z^2}{(z-1)^3}.$$

5 Residue Theorem

1. Use the Residue Theorem to compute
$$\int_{\infty}^{\infty} \frac{1}{x^2 + 4} \, dx$$
Compare your result with the result obtained using well-known integration formula.

2. Compute the contour integral:
$$\oint_{|z|=3} \frac{(z-1)^3}{z(z+2)^3} \, dz.$$