(1) (Evans, pg. 85 #1). Find an explicit formula for the solution $u : \mathbb{R}^n \to \mathbb{R}$ of the initial value problem:

$$u_t + b \cdot Du + cu = 0, \quad (x, t) \in \mathbb{R}^n \times (0, \infty)$$

$$u(x, t = 0) = g(x), \quad (x, t = 0) \in \mathbb{R}^n \times \{0\}. \quad (1)$$

Here, the vector $b \in \mathbb{R}^n$ and the number $c \in \mathbb{R}$ are constants.

(2) (Evans, pg. 85, #2). Suppose that $u(x)$ is a solution to Laplace’s equation $\Delta u = 0$ on $\mathbb{R}^n$. Prove that if $O$ is an orthogonal transformation, then the function $v(x) \equiv u(Ox)$ is also a solution to Laplace’s equation.

(3) Let $\Omega \subset \mathbb{R}^n$ be a bounded open set with $C^1$-boundary. Prove the following identities using the Gauss-Green Theorem:

- Integration by parts: For any $u, v \in C^1(\Omega)$, we have

$$\int_\Omega u_j v = - \int_\Omega u v_j + \int_{\partial \Omega} u v \nu_j, \quad (2)$$

where $\nu(x)$ is the outward normal vector at $x \in \partial \Omega$.

- Green’s formula: For any $u, v \in C^2(\Omega)$, we have

$$\int_\Omega u \Delta v = - \int_\Omega \nabla u \cdot \nabla v + \int_{\partial \Omega} (\nu \cdot \nabla v) u. \quad (3)$$

- For any $u, v \in C^2(\Omega)$, we have

$$\int_\Omega [u \Delta v - v \Delta u] = \int_{\partial \Omega} [u(\nu \cdot \nabla v) - v(\nu \cdot \nabla u)]. \quad (4)$$