(1a) \( \{x_i\} \) bounded non-decr. \( \Rightarrow \) the set is bounded above so \( \sup \{x_i\} \) exists.
For any \( \varepsilon > 0 \), \( \exists n_\varepsilon \) s.t. \( a - \varepsilon \leq x_{n_\varepsilon} \leq a \).
Otherwise, \( a \) would not be an upper bound.

Let \( \varepsilon = \frac{1}{k} \), \( k = 1, 2, 3, \ldots \) and choose \( x_k \) so \( a - \frac{1}{k} \leq x_k \leq a \).

Since \( x_m \geq x_k \) when \( m \geq k \) and \( x_m \leq a \),
we have \( |x_m - a| < \frac{1}{k} \) for \( m > k \).

This proves \( x_m \to a \).

(1b) We show \( \inf A \) is a limit point.

Apply the same construction. For each \( k \), \( \exists x_k \) so \( a - \frac{1}{k} \leq x_k \leq a \).

This means each \( N_{\frac{1}{k}}(a) \) contains a point of \( A \) so as \( A \) is closed, \( a \in A \).
(2a) \implies \text{Suppose } f : S \to T \text{ is continuous. Let } K \subset T \text{ be closed so } K^c \subset T \text{ is open. We know } f^{-1}(K^c) \subset S \text{ is open since } f \text{ is continuous.}

Claim: \quad f^{-1}(K^c)^c = f^{-1}(K)

Given this, \quad f^{-1}(K)^c \text{ is closed so } f^{-1}(K) \text{ is closed.}

\textbf{Proof of Claim:} \quad \text{If } y \notin f^{-1}(K), \; f(y) \notin K

\text{so } f(y) \notin K \implies y \notin f^{-1}(K)

\text{so } \quad (f^{-1}(K))^c \subset f^{-1}(K)

Conversely, \quad \text{If } w \in f^{-1}(K), \; f(w) \in K \text{ so } w \in f^{-1}(K) \text{ so } f^{-1}(K) \subset (f^{-1}(K))^c \quad \square

\Leftrightarrow \quad \text{Suppose } K \subset T \text{ is closed, } f^{-1}(K) \subset S \text{ is closed. Let } \emptyset \subset T \text{ be open so } \emptyset^c \subset T \text{ is closed}

f^{-1}(\emptyset^c) \subset S \text{ is closed and } (f^{-1}(\emptyset^c))^c \subset S \text{ is open. By the claim above, } \quad (f^{-1}(\emptyset^c))^c = f^{-1}(\emptyset) \text{ is open } \quad \square

(2b) \quad f : S \to T \text{ is bijective } \& S \text{ compact } \iff f^{-1} : T \to S \text{ exists. It is continuous if } K \subset S \text{ is closed, } \quad (f^{-1})^{-1}(K) \subset T \text{ is closed. But } (f^{-1})^{-1} = f \text{ so we must show } f(K) \text{ is closed. As } S \text{ is compact, } K \subset S \text{ closed is also compact. As } f \text{ is continuous, } f(K) \subset T \text{ is compact and thus closed. } \quad \square
(3a) A seq. \( \{x_n\} \) is Cauchy if \( \forall \varepsilon > 0 \ \exists N \in \mathbb{N} \) so
\[ n, m > N \Rightarrow d_S(x_n, x_m) < \varepsilon. \]

(3b) Suppose \( \{x_{n_j}\} \) is a subseq. of \( \{x_n\} \), that \( \{x_n\} \) is Cauchy and \( \lim_{j \to \infty} x_{n_j} = x_0 \) exists.
\[ j \in \mathbb{N} \Rightarrow n_j \in \mathbb{N} \text{ is increasing } n_j > j. \]
\[ \text{Given } \varepsilon > 0, \exists J_\varepsilon \text{ s.t. } j > J_\varepsilon \Rightarrow d_S(x_{n_j}, x_0) < \varepsilon/2, \]
\[ \exists N_\varepsilon \text{ s.t. } n > N_\varepsilon \Rightarrow d_S(x_n, x_0) < \varepsilon/2. \]

Set \( N_\varepsilon = \max(J_\varepsilon, N_\varepsilon) \). Then \( j > N_\varepsilon \Rightarrow \]
\[ d_S(x_n, x_0) \leq d_S(x_n, x_{n_j}) + d_S(x_{n_j}, x_0) \]
Choosing \( j > N_\varepsilon \Rightarrow n_j > N_\varepsilon \) (Cauchy cond.)
and \( j > J_\varepsilon \) (conv. of subseq. cond.), we get
\[ d_S(x_n, x_0) \leq \varepsilon/2 + \varepsilon/2 = \varepsilon. \]
so \( x_n \to x_0 \) in \( S \). \( \blacksquare \)
(4.) A compact metric space is complete.

Let \( \{x_j\} \) be Cauchy in \((S,d_s)\). If not convergent, no \( p \) is a limit point of the set \( \{x_j\} \).

Every point \( p \in S \) lies in \( \bigcap_{\epsilon > 0} B_{\epsilon}(p) \) containing at most finitely many points of \( \{x_j\} \).

\[
S = \bigcup_{p \in S} B_{\epsilon}(p) \quad \text{since } S \text{ is compact}
\]

\[
S = \bigcup_{p \in S} B_{\epsilon}(p) \quad \text{then } \{x_j\} \text{ has finitely many elements & } d(x_j, x_k) > \epsilon \quad \forall j, k
\]

So can't be Cauchy.