MA641 Differential Geometry
Spring 2014
Problem Set 3
DUE: Friday, 7 March 2014

1. do Carmo: page 32, exercise 2 on the orientability of the tangent bundle.

2. Prove that a regular surface $S \subset R^3$ is orientable if and only if there is a differentiable map (a normal vector to the surface) $N : S \to R^3$ so that $N(p)$ is orthogonal to T_pS and $\|N(p)\| = 1$ for all $p \in S$.

3. Consider the following parametrization of the Möbius strip considered as an embedded surface $S \subset R^3$:

$$(u, v) \in U = (0, 2\pi) \times (-1, 1) \subset R^2 \to \begin{pmatrix} (1 + v\sin(u/2))\cos u \\ (1 + v\sin(u/2))\sin u \\ v\cos(u/2) \end{pmatrix}.$$

   Using the criterion in problem 1, show that the Möbius strip is non-orientable. HINT: Study a normal vector at $(0, 0)$ and at $(2\pi, 0)$.

4. do Carmo, page 46, #4, part (a). As a set, the group G is $G = \{(x, y) \mid y > 0, x \in R\}$. Group multiplication on G is obtained by composition of affine functions. Let $g_{x,y}(t) = yt + x$, with $y > 0$, and $x \in R$. Compute the group law by computing $g_{x,y} \circ g_{u,v}$. Compute the identity element and the inverse. Writing elements of G as (x, y), with $y > 0$, the group law can be written as $(x, y) \cdot (u, v) = (??, ??)$, where the second component is positive. With this identification, we can map $R^2_+ = \{(x, y) \mid y > 0\}$ into G by $\phi(x, y) = g_{x,y}$. Then (ϕ, R^2_+) is a globally defined parametrization of G. Compute the group operations in the (x, y) parametrization. The left action of G on itself can be written as $\phi^{-1} \circ L_{g_{x,y}} \circ \phi$. Write this out explicitly and compute the differential. Using $\partial/\partial x$ and $\partial/\partial y$ as a basis for the tangent space write out the condition that the metric be left invariant. Now complete part (a) of the problem.