MA641 Differential Geometry
Spring 2014
Problem Set 4
DUE: Friday, 28 March 2014

(1) do Carmo, page 56, #1, on parallel transport.
(2) do Carmo, page 57, #4, on parallel transport for regular surfaces.
(3) Prove that $SU(2)$ is simply connected and homeomorphic to the three sphere S^3 using the map:
$$x \in S^3 \to x_1 I_2 + i(x_2, x_3, x_4) \cdot \overline{\sigma},$$
where $\overline{\sigma}$ is the vector formed from the three Pauli matrices.
(4) Continuation of do Carmo, page 46, #4: Prove that the mappings
$$\alpha_A(z) = \frac{az+b}{cz+d}$$
for
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,R),$$
are isometries of the left-invariant Riemannian metric constructed in the problem.