MA641 Differential Geometry
Fall 2010
Problem Set 1
August 27, 2010
DUE: Friday, 10 September

1. Prove that the circle S^1 is a differentiable manifold using the stereographic projection.

2. Complete the proof (sketched on page 3 of do Carmo) that a differentiable manifold is a topological space, that the sets $x_\alpha(U)$ are open, and that the maps x_α are continuous.

3. Suppose that $\gamma : I \subset R \to R^n$ is a differentiable curve defined for the interval I, and that $f : U \subset R^n \to R$ is a C^1-function defined on an open set U containing $\gamma(I)$. Show that for any $t \in I$,

$$\frac{d}{dt}(f \circ \gamma)(t) = (\nabla f)(\gamma(t)) \cdot \gamma'(t),$$

   where ∇f is the gradient of f and $\gamma'(t)$ is the tangent vector to the curve γ at t.

4. The sphere $S^{n-1} \subset R^n$ can be realized as the zero set of the function

$$F(x, \ldots, x_n) = \sum_{i=1}^{n} x_i^2 - 1.$$

   Suppose that $\gamma : I \subset R \to R^n$ is a differentiable curve defined for the interval I that obeys the equation

$$F(\gamma(t)) = 0, \quad \forall t \in I.$$

   Prove that

$$\gamma(t) \cdot \gamma'(t) = 0, \quad \forall t \in I.$$

   What does this condition mean geometrically?