MA641 Differential Geometry
Fall 2010
Problem Set 2
DUE: Friday, 24 September

1. Let $S \subset R^3$ be a 2 dimensional regular surface. Prove that locally S is the graph of a smooth function. That is, for any $p \in S$, there is a neighborhood $V_p \subset S$ of p so that V is the graph of a smooth function that has one of the following three forms: $z = f(x, y)$, or $y = g(x, z)$, or $x = h(y, z)$. *Hint:* there is a coordinate map $\mathbf{x} : U \subset R^2 \to R^3$ with $\mathbf{x}(0) = p$ and $d\mathbf{x}_0$ has maximal rank.

2. Consider local parameterizations (ϕ_α, U_α) of the 2-sphere S^2 of the form $(u, v, \sqrt{1 - u^2 - v^2})$. Prove that S^2 is a differentiable manifold using this atlas. This requires that one check
   (a) $f_\alpha : U_\alpha \to S^2$ is a homeomorphism,
   (b) df_α has maximal rank,
   (c) $\cup_\alpha U_\alpha = S^2$,
   (d) the overlap maps $f_\beta^{-1} \cdot f_\alpha$ are smooth (give a formula).

3. Let $\gamma(t)$ be the parameterized curve:
$$\gamma(t) = (a \cos(t/c), a \sin(t/c), bt/c),$$
   for $s \in R$ and positive real numbers (a, b, c) so that $a^2 + b^2 = c^2$.
   (a) Write the curve in arc length parametrization.
   (b) Compute the curvature and torsion.
   (c) Show that the lines tangent to γ make a constant angle with the z-axis.

4. do Carmo: page 32, exercise 2 on the orientability of the tangent bundle.