

MA641 Differential Geometry
Fall 2010
Problem Set 3
DUE: Friday, 8 October 2010

1. Prove that a regular surface $S \subset R^3$ is orientable if and only if there is a differentiable map (a normal vector to the surface) $N : S \rightarrow R^3$ so that $N(p)$ is orthogonal to $T_p S$ and $\|N(p)\| = 1$ for all $p \in S$.
2. Consider the following parametrization of the Möbius strip considered as an embedded surface $S \subset R^3$:

$$(u, v) \in U = (0, 2\pi) \times (-1, 1) \subset R^2 \rightarrow \begin{pmatrix} (1 + v \sin(u/2)) \cos u \\ (1 + v \sin(u/2)) \sin u \\ v \cos(u/2) \end{pmatrix}.$$

Using the criterion in problem 1, show that the Möbius strip is non-orientable. HINT: Study a normal vector at $(0, 0)$ and at $(2\pi, 0)$.

3. do Carmo, page 46, #4: The group law on G is composition of functions. For $g_{x,y}(t) = yt + x$, with $y > 0$, and $f_{u,v}(t) = vt + u$, with $v > 0$, compute $(f \circ g)(t)$. This can be written as $(x, y) \cdot (u, v) = (??, ??)$ where the second component is positive. With this identification, we can map $R_+^2 = \{(x, y) \mid y > 0\}$ into G by $\phi(x, y) = g_{x,y}$. Then (ϕ, R_+^2) is a globally defined parametrization of G . Compute the group operations in the (x, y) parametrization. The left and right actions of G on itself can be written as $\phi^{-1} \circ L_{g_{x,y}} \circ \phi$. Write this out explicitly and compute the differential. Using $\partial/\partial x$ and $\partial/\partial y$ as a basis for the tangent space write out the condition that the metric be left invariant. Now complete the problem.