

Table 4.1.1 Techniques for Finding Residues

In this table g and h are analytic at z_0 and f has an isolated singularity. The most useful and common tests are indicated by an asterisk.

Function	Test	Type of Singularity	Residue at z_0
1. $f(z)$	$\lim_{z \rightarrow z_0} (z - z_0) f(z) = 0$	removable	0
*2. $\frac{g(z)}{h(z)}$	g and h have zeros of same order	removable	0
*3. $f(z)$	$\lim_{z \rightarrow z_0} (z - z_0) f(z) = 0$ exists and is $\neq 0$	simple pole	$\lim_{z \rightarrow z_0} (z - z_0) f(z)$
*4. $\frac{g(z)}{h(z)}$	$g(z_0) \neq 0, h(z_0) = 0,$ $h'(z_0) \neq 0$	simple pole	$\frac{g(z_0)}{h'(z_0)}$
5. $\frac{g(z)}{h(z)}$	g has zero of order k , h has zero of order $k+1$	simple pole	$(k+1) \frac{g^{(k)}(z_0)}{h^{(k+1)}(z_0)}$
*6. $\frac{g(z)}{h(z)}$	$g(z_0) \neq 0$ $h(z_0) = 0 = h'(z_0)$ $h''(z_0) \neq 0$	second-order pole	$\frac{2}{h''(z_0)} \frac{g'(z_0)}{3} - \frac{2}{[h''(z_0)]^2} \frac{g(z_0)h'''(z_0)}{3}$
*7. $\frac{g(z)}{(z - z_0)^2}$	$g(z_0) \neq 0$	second-order pole	$g'(z_0)$
*8. $\frac{g(z)}{h(z)}$	$g(z_0) = 0, g'(z_0) \neq 0,$ $h(z_0) = 0 = h'(z_0)$ $= h''(z_0), h'''(z_0) \neq 0$	second-order pole	$\frac{3}{h'''(z_0)} \frac{g''(z_0)}{2} - \frac{3}{[h''(z_0)]^2} \frac{g'(z_0)h^{(iv)}(z_0)}{2}$
9. $f(z)$	$\left\{ \begin{array}{l} \text{that } \lim_{z \rightarrow z_0} \phi(z) \text{ exists where} \\ \phi(z) = (z - z_0)^k f(z) \end{array} \right.$	pole of order k	$\lim_{z \rightarrow z_0} \frac{\phi^{(k-1)}(z)}{(k-1)!}$
*10. $\frac{g(z)}{h(z)}$	g has zero of order l , h has zero of order $k+l$	pole of order k	$\lim_{z \rightarrow z_0} \frac{\phi^{(k-1)}(z)}{(k-1)!}$ where $\phi(z) = (z - z_0)^k \frac{g}{h}$
11. $\frac{g(z)}{h(z)}$	$g(z_0) \neq 0, h(z_0) =$ $\dots = h^{k-1}(z_0)$ $= 0, h^k(z_0) \neq 0$	see Proposition 4.1.7.	