
2. Harmonic functions and harmonic extensions.
   (a) Stein and Shakarchi, page 66: Problem 11.
   (b) Stein and Shakarchi, page 67: Problem 12 b (we already did part a).
   (c) A comment: Suppose \( f(\varphi) \) is a continuous (therefore periodic) function on the unit circle. Then \( u_f(z) \), with \( z = re^{-\theta} \), \( 0 \leq r < 1 \), defined on the unit disc by

   \[
   u_f(z) = \frac{1}{2\pi} \int_0^{2\pi} P_r(\theta - \varphi) f(\varphi) \, d\varphi
   \]

   is harmonic and satisfies the boundary condition \( u_f|_C = f \). This function \( u_f \) is called the harmonic extension of \( f \) to the disc (nothing for you to compute here).
