MA676 Spring 2009
Homework Problem Set #1
January 16, 2009

Assignment 1. Read the Introduction of Stein-Shakarchi for culture and enlightenment. Begin reading chapter 1 sections 1-3. These problems are due Monday, 26 January 2009. (WZ means the problems are from Wheeden-Zygmund).

1. Prove that our favorite function $f : [0,1] \to \{0,1\}$ defined by $f(x) = 1$, for $x \in Q \cap [0,1]$ and $f(x) = 0$ otherwise is not continuous at any point in $[0,1]$.

2. Complete the proof of the representation of any open set $\mathcal{O} \subset \mathcal{R}$ as a countable union of disjoint open intervals by showing that the representation is unique.

3. (WZ page 13) Prove that the distance between two nonempty compact disjoint set in R^d is positive.

4. (WZ page 12) Let E_j be a countable collections of sets $E_j \subset R^d$, and define $\limsup E_k \equiv \cap_{j=1}^{\infty} \left(\cup_{k=j}^{\infty} E_k \right)$. Prove that$\limsup E_k$ consists of those points which belong to infinitely many E_j. Formulate a similar result for $\liminf E_k$.

5. (WZ page 13) If $E_k = [-(1/k), 1]$ for k odd, and $E_k = [-1, (1/k)]$ for k even, find $\limsup E_k$ and $\liminf E_k$.

6. (WZ page 13) Show that the intersection of a countable sequence of decreasing nonempty compact sets is nonempty.

7. (WZ page 13) Give an example of a decreasing sequence of nonempty closed sets in R^d whose intersection is empty