

MA676 Spring 2009
Homework Problem Set #3
February 11, 2009

Assignment 3. Read section 4 of Chapter 1 on measurable functions in Stein-Shakarchi. These problems are due Monday, 16 February 2009. Problem discussion Friday, 13 Feb. at 4PM. (WZ means the problems are from Wheeden-Zygmund).

- (1) Prove that for any $A \subset \mathbb{R}^d$ there is a G_δ set G with $A \subset G$ and $m_* A = m_* G$.
- (2) S^2 , page 37: Problems 1, 2, and 3 on Cantor sets and the Cantor function.
- (3) S^2 , page 43: Problems 25, 26 on measurability.
- (4) (W-Z, page 48) Define the inner measure by $m_i(A) = \sup m(F)$, where the supremum is taken over all closed sets $F \subset A$. Show that $m_i(A) \leq m_*(A)$ and that if $m_*(A) < \infty$, then A is measurable if and only if $m_i(A) = m_*(A)$.