

**MA676 Spring 2009**  
**Homework Problem Set #4**  
**February 20, 2009**

Assignment 3. Read section 1 of Chapter 2 on integration of measurable functions in Stein-Shakarchi. These problems are due Wednesday, 4 March 2009. Problem discussion Friday, 27 Feb. at 4PM. ( WZ means the problems are from Wheeden-Zygmund).

- (1) Prove Property 6 on page 30 of  $S^2$ .
- (2)  $S^2$ , page 42, problems 17 (we already did problem 16, the Borel-Cantelli lemma.)
- (3)  $S^2$ , page 89, problem 1. Use this to prove Proposition 1.1, properties (ii) and (iv).
- (4)  $S^2$ , page 91, problem 9,
- (5) Suppose that  $f$  is a bounded, measurable function on a measurable set  $E$  of finite measure. Prove that

$$\int f = \sup_{\phi \leq f} \int \phi = \inf_{\psi \geq f} \int \psi,$$

for simple functions  $\phi$  and  $\psi$ . These integrals provide an alternate definition to the one in  $S^2$ . HINT: first consider  $f$  with a fixed sign.

- (6) Use problems 1 and 5 to show that if for a bounded function  $f$  on a measurable set  $E$  of finite measure we have

$$\sup_{\phi \leq f} \int \phi = \inf_{\psi \geq f} \int \psi,$$

then  $f$  is measurable on  $E$ .

- (7) Show that the Monotone Convergence Theorem does not hold for a monotone decreasing sequence of nonnegative measurable functions.