These problems are on the material in chapter 3 of Stein-Shakarchi. These problems are due Friday, 1 May. We’ll discuss them during the last week of class. (WZ means the problems are from Wheeden-Zygmund).

1. Prove that if $f \in L^1([a,b])$, then $F(x) = \int_a^x f \in AC[a,b]$.

2. $S^2$, page 147, problem 14 on the measurability of certain difference quotients.

3. $S^2$, page 95, problem 3 on convergence in measure.

4. ((WZ) page 85, # 10.) If $p > 0$, and $f_k$ is a sequence of integrable functions with $\int_E |f_k - f|^p \to 0$ (as in the problem above) and there exists a constant $M > 0$ so $\int_E |f_k|^p < M < \infty$, for all $k$, then $\int_E |f|^p < M$. Use both Fatou’s lemma and the existence of a pointwise convergent subsequence.