

MA676 Spring 2009
Homework Problem Set #7
Due: Friday, 1 May 2009
April 17, 2009

These problems are on the material in chapter 3 of Stein-Shakarchi. These problems are due Friday, 1 May. We'll discuss them during the last week of class. (WZ means the problems are from Wheeden-Zygmund).

1. Prove that if $f \in L^1([a, b])$, then $F(x) = \int_a^x f \in AC[a, b]$.
2. S^2 , page 147, problem 14 on the measurability of certain difference quotients.
3. S^2 , page 95, problem 3 on convergence in measure.
4. ((WZ) page 85, # 10.) If $p > 0$, and f_k is a sequence of integrable functions with $\int_E |f_k - f|^p \rightarrow 0$ (as in the problem above) and there exists a constant $M > 0$ so $\int_E |f_k|^p < M < \infty$, for all k , then $\int_E |f|^p < M$. Use both Fatou's lemma and the existence of a pointwise convergent subsequence.