Assignment 2. Finish reading chapter 4 of Stein-Shakarchi on Hilbert space theory. These problems on pages 193-194 are due Friday, 2 October 2009.

(1) Give a complete proof that a pre-Hilbert space (an inner product space) has a completion. That is, prove that there exists a Hilbert space so that the pre-Hilbert space is isometrically isomorphic with a dense subset of the Hilbert space.

(2) Problem 9 on the construction of an ONB for $L^2(\mathbb{R})$.

(3) Problem 10 on orthogonal complements.

(4) Problem 11 on orthogonal projectors.

(5) Problem 12. This is an extension of problem 11 in a specific setting. Make sure you prove that $\mathcal{S}$ is closed.