

MA681–001 Functional Analysis
Fall 2016
Problem Set 3
DUE: Monday, 3 October 2016

1. Read pages 305–306 of Hislop-Sigal on the set of all bounded operators from X to Y , which is denoted $\mathcal{L}(X, Y)$ (also called $\mathcal{B}(X, Y)$). Do problem A3.7, which finishes the proof of Theorem A3.16. When we have $X = Y$ and use the adjoint, show that $\mathcal{L}(X)$ is a *Banach*algebra*.
2. Consider the Banach space $(C([0, 1]), \|\cdot\|_\infty)$. Let $a \in C([0, 1])$ and define the multiplication operator $A : f \in C([0, 1]) \rightarrow af$. Show that A is a bounded linear operator and compute its norm. If $|a| < 1$, show that the operator $1 + A$ is a boundedly invertible operator.
3. Conway, page 36: # 7, 12 and 14, 15.
4. Conway, page 40: # 1, 4.