MA 681 001 Fall 2016

Functional Analysis

P. D. Hislop

MWF 2:00-2:50 CB 347

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The purpose of this course is to present some basic elements of Banach and Hilbert space theory and the theory of linear operators. The main goal will be the spectral theory of linear operators including the functional calculus. We will apply the theory to study the basic properties of the Laplacian on bounded and unbounded subsets of R^n, and other spaces of constant curvature, and the basic spectral properties of Schrödinger operators. We will cover global Sobolev space theory and Fourier transforms. We will also discuss some basic methods in nonlinear theory, such as the contraction mapping principle, and apply it to prove the existence of solutions to some nonlinear equations such as the Euler equation.

1. Introduction to Banach and Hilbert spaces
2. Linear operators: spectrum and resolvent
3. Self-adjoint operators
4. Banach space theory: Hahn-Banach Theorem, Closed Graph Theorem, etc.
5. Compact Operators and Fredholm theory
6. Spectral theory of linear operators
7. Analytic Perturbation Theory
8. Spectral Stability
9. Functional Calculus
10. Special Topic: Laplacians on Bounded Domains and Eigenvalue Asymptotics
11. Special Topic: Contraction Mapping Principle and Nonlinear Problems

There is no required text for the course. I will follow chapters from John Conway's book *A course in functional analysis, 2nd edition*, Springer, 1990, Graduate Texts in Mathematics vol. 96, and from my book with I. M. Sigal, *Introduction to Spectral Analysis, with Applications to Schrödinger Operators*, Applied Mathematical Sciences volume 113, Springer, 1996.

Other text closely related to the course include

1. M. Reed and B. Simon, *Methods of Modern Mathematical Physics*, volumes 1–4 (especially volume 1), Academic Press.
2. E. B. Davies, *Spectral theory and Differential Operators*, Cambridge University Press, 1995.
3. M. Demuth, M. Krishna, *Determining Spectra in Quantum Theory*, Birkhaüser, 2005.
4. R. Courant, D. Hilbert, *Methods of Mathematical Physics*, Vol. 1 and 2, Wiley Classics (reprinted), 1989.
5. P. Halmos, *Introduction to Hilbert Space*, Chelsea, 1957. (The first two chapters are especially good).
6. G. Folland, *Introduction to Partial Differential Equations*, Princeton, 1995 (Chapters 2, 4, 5, and 6 are especially relevant to our course).

Course Requirements:
The course is self-contained. Everyone should be comfortable with the basic arguments of analysis as taught in the Rudin course MA575. A PDE course or a measure theory course is not a prerequisite but useful. Each student will choose a research topic in consultation with the instructor and write a short paper (approximately 10-15 pages) using LaTex on the topic. Each student will make an oral presentation to the class towards the end of the semester. There will be approximately eight problem sets during the semester.