**MA681–001 Functional Analysis**
**Fall 2019**
**Problem Set 2**
**DUE: Wednesday, 18 September 2019**

1. Read pages 305–306 of H-S on the set of all bounded operators from $X$ to $Y$, which is denoted $\mathcal{L}(X, Y)$. Do problem A3.7, which finishes the proof that this algebra is a Banach space. When we have $X = Y$, show that $\mathcal{L}(X)$ is a Banach algebra.

2. Let $(X, \| \cdot \|)$ be a NLVS. A sequence $\{x_j\} \subset X$ is absolutely summable if $\sum_j \|x_j\| < \infty$ and summable if the sequence of partial sums $\left\{\sum_{j=1}^N x_j\right\}$ converges in $X$. Prove: The NLVS $(X, \| \cdot \|)$ is complete if and only if every absolutely summable sequence is summable.

3. Consider the Banach space $(C([0,1]), \| \cdot \|_\infty)$. Let $a \in C([0,1])$ and define the multiplication operator $A : f \in C([0,1]) \to af$. Show that $A$ is a bounded linear operator and compute it's norm. If $\|a\|_\infty < 1$, show that the operator $1 + A$ is a boundedly invertible operator.

4. Prove that the minimizer $m_0 \in \mathcal{M}$ of the distance functional $\text{dist}(h, \mathcal{M})$, for fixed $h \in \mathcal{H}$, is unique.