MA681–001 Functional Analysis
Fall 2019
Problem Set 3
DUE: Wednesday, 2 October 2019

1. Conway, page 6: #3, and page 13: #5.
2. Conway, page 18: # 14.
3. Conway, page 18: #16. A Hamel basis of a LVS is a set of linearly independent vectors so that each vector in the LVS is a finite linear combination of elements in the set.
4. A nonseparable Hilbert space. Consider $C(R)$ with the seminorm

$$\|f\|_0 := \left(\lim_{N \to \infty} \frac{1}{2N} \int_{-N}^{N} |f(x)|^2 \, dx \right)^{\frac{1}{2}}.$$

Of course, any $L^2(R)$-function has seminorm zero. Let $\mathcal{H}_\prime$ be the span (all finite linear combinations) of the functions $e^{i\lambda x}$, for any $\lambda \in R$. Then show that $(\mathcal{H}_\prime, \| \cdot \|_\prime)$ is an IPS. Let $mathcalH$ be the completion of this space. Prove that $\mathcal{H}$ is a nonseparable Hilbert space.