MA681–001 Functional Analysis
Fall 2019
Problem Set 4
DUE: Monday, 14 October 2019

(1) Prove that the identity operator $I \in \mathcal{B}(\mathcal{H})$ is compact if and only if the Hilbert space $\mathcal{H}$ is finite dimensional.

(2) Prove that if a compact operator $T \in \mathcal{K}(\mathcal{H})$ is invertible (that is, $0 \notin \sigma(T)$), then the Hilbert space $\mathcal{H}$ is finite dimensional.

(3) Prove that an isometry on a finite-dimensional complex Hilbert space (isomorphic to $\mathbb{C}^N$, $N < \infty$) is unitary.

(4) Let T_L be the left shift operator on $\ell^2_{\mathbb{C}}(\mathbb{N})$. Compute the spectrum of T_L. Find the eigenvalues of T_L and the corresponding eigenvectors. What can you say about the spectrum of its adjoint? Does the adjoint have any eigenvalues?

(5) Suppose T_n is a sequence of bounded operators on a Hilbert space $\mathcal{H}$ so that $\langle f, T_n g\rangle$ converges for each pair of vectors $f, g \in \mathcal{H}$. Prove that there exists a bounded operator T so that the sequence T_n converges weakly to T. Hint: use the principle of uniform boundedness twice.