MA681–001 Functional Analysis
Fall 2019
Problem Set 5
DUE: Wednesday, 9 November 2019

(1) Let X be a nontrivial normed linear vector space and fix a nonzero $w \in X$. Then there is a nonzero bounded linear functional Λ on X so that $\Lambda(w) = \|w\|$ and $\|\Lambda\|_{X^\star} = 1$.
(2) Use the Baire Category Theorem to prove that any Hamel basis of a non-finite dimensional Banach space must be uncountable.
(3) Let X be a Banach space. Prove that a weakly open set is norm open and that a weakly convergent sequence is norm bounded.
(4) Let $X = C([0,1])$ with norm $\|f\|_1 := \int_0^1 |f(s)|\, ds$ be a NLVS. Prove that the linear functional $L(f) = f(\frac{1}{2})$ isn't bounded.