MA 114: Calculus II
Exam 2: Extra Credit

Directions: The following problems have been selected as opportunities to earn some additional points toward your Exam 2 score. Complete solutions should be written on another sheet of paper and turned in at the beginning of class on Monday, July 27, 2015. You must show your work to receive any credit. You may work with other students and consult your textbook and/or notes, but no additional resources should be used.

1. (2 points) Find the fourth Taylor polynomial for \( f(x) = x^6 - 5x^4 + 9x^2 - 3 \) centered at \( x = -1 \). You must simplify your answer.

2. (3 points) Consider the region in the first quadrant enclosed by the graphs of \( f(x) = \frac{9}{x} \) and \( g(x) = 12 - 3x \). Draw a clear picture of the region in question, and set up and evaluate an appropriate integral to calculate the volume of the solid obtained by rotating this region around the \( y \)-axis.

   Note: If you choose to use the Disk/Washer method, be sure to indicate the relevant radius/radii on your picture of the region. If you choose to use the Shell method, be sure to include a representative rectangle in your picture.

3. (3 points) Evaluate the indefinite integral \( \int x^3(x^2 + 4)^{3/2} \, dx \). Be sure to properly identify any substitutions that you use.
1.) Find fourth Taylor polynomial for \( f(x) = x^6 - 5x^4 + 9x^2 - 3 \) centered at \( x = -1 \).

\[
\begin{align*}
  f'(x) &= 6x^5 - 20x^3 + 18x \\
  f''(x) &= 30x^4 - 60x^2 + 18 \\
  f'''(x) &= 120x^3 - 120x \\
  f^{(4)}(x) &= 360x^2 - 120 \\
  f(-1) &= -1 - (-5) - 9 - 3 = 2 \\
  f''(-1) &= -6 + 20 - 18 = -4 \\
  f'''(-1) &= -120 + 120 = 0 \\
  f^{(4)}(-1) &= 360 - 120 = 240
\end{align*}
\]

\[
T_4(x) = 2 + \frac{-4}{1!}(x+1) + \frac{-12}{2!}(x+1)^2 + \frac{0}{3!}(x+1)^3 + \frac{240}{4!}(x+1)^4
\]

\[
= 2 - 4(x+1) - 6(x^2 + 2x + 1) + 10(x^4 + 4x^3 + 6x^2 + 4x + 1)
\]

\[
= \frac{10x^4 + 40x^3 + (-6 + 60)x^2 + (-4 - 12 + 40)x + (2 - 4 - 6 + 10)}{10x^4 + 40x^3 + 54x^2 + 24x + 2}
\]

2.) \( f(x) = \frac{9}{x} \)

\( g(x) = 12 - 3x \)

**Intersection Points**

\[
\frac{9}{x} = 12 - 3x \Rightarrow \]

\[
9 = 12x - 3x^2 \Rightarrow
\]

\[
3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 0 \]

\[
= 3(x - 3)(x - 1) \quad \text{radius: } x
\]

\[
x = 1, 3 \quad \text{height: } 12 - 3x - \frac{9}{x}
\]

\[
V = 2\pi \int_1^3 x \left(12 - 3x - \frac{9}{x}\right) \, dx \quad \text{(Shell Method)}
\]
2. (cont.) \[ V = 2\pi \int_1^3 x \left(12 - 3x - \frac{9}{x}\right) \, dx \]

\[ = 2\pi \int_1^3 12x - 3x^2 - 9 \, dx \]

\[ = 2\pi \left[ \left(6x^2 - x^3 - 9x\right) \right]_1^3 \]

\[ = 2\pi \left[ (54 - 27 - 27) - (6 - 1 - 9) \right] = 8\pi \]

\[ q(x) = 12 - 3x \quad y = 12 - 3x \Rightarrow \quad x = \frac{12 - y}{3} = 4 - \frac{1}{3} y \]

\[ f(x) = 9x \quad y = \frac{9}{x} \Rightarrow \quad x = \frac{9}{y} \]

\( R = 4 - \frac{1}{3} y, \quad r = \frac{9}{y} \)

(Washer Method)

\[ V = \pi \int_3^9 \left(4 - \frac{1}{3} y\right)^2 - \left(\frac{9}{y}\right)^2 \, dy \]

\[ = \pi \int_3^9 \left(16 - \frac{8}{3} y + \frac{1}{9} y^2\right) - \left(\frac{81}{y^2}\right) \, dy \]

\[ = \pi \left[ \left(16y - \frac{8}{3} y^2 + \frac{1}{2} y^3 + \frac{81}{y}\right) \right]_3^9 \]

\[ = \pi \left[ (144 - 324 + \frac{729}{27} + 9) - (48 - \frac{36}{3} + \frac{27}{27} + \frac{81}{3}) \right] \]

\[ = \pi (72 - 64) = 8\pi \]
\[ \int x^3 (x^2 + 4)^{3/2} \, dx \]

Let \( x = 2 \tan \theta \),
\[ dx = 2 \sec^2 \theta \, d\theta \]
\[ \sqrt{x^2 + 4} = 2 \sec \theta \]
\[ (2 \tan \theta)^3 (2 \sec \theta)^3 (2 \sec^2 \theta) \, d\theta \]
\[ = \frac{1}{2} 8 \tan^3 \theta \cdot 8 \sec^3 \theta \cdot 2 \sec^2 \theta \, d\theta \]
\[ = 128 \int \tan^3 \theta \sec^5 \theta \, d\theta = 128 \int \tan^2 \theta \sec^3 \theta (\sec \theta \tan \theta \, d\theta) \]
\[ = 128 \int (\sec^2 \theta - 1) \sec^4 \theta (\sec \theta \tan \theta \, d\theta) \]
\[ = 128 \int (u^2 - 1) u^4 \, du \quad \left\{ \begin{array}{l} u = \sec \theta \\ du = \sec \theta \tan \theta \, d\theta \end{array} \right. \]
\[ = 128 \int u^6 - u^4 \, du = 128 \left[ \frac{1}{6} u^6 - \frac{1}{4} u^4 \right] + C \]
\[ = \frac{128}{7} \sec^7 \theta - \frac{128}{5} \sec^5 \theta + C \]
\[ = \frac{128}{7} \left( \frac{\sqrt{x^2 + 4}}{2} \right)^7 - \frac{128}{5} \left( \frac{\sqrt{x^2 + 4}}{2} \right)^5 + C \]