1. Let \( f(x) = \frac{|3x - 18|}{12 - 2x} \). Complete the following table chart to find \( \lim_{x \to 6} f(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>5.5</th>
<th>5.9</th>
<th>5.99</th>
<th>6.01</th>
<th>6.1</th>
<th>6.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>-1.5</td>
<td>-1.5</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

Since the limit from the left-hand side (1.5) and the limit from right-hand side (-1.5) do not match, \( \lim_{x \to 6} f(x) \) does not exist.

2. Suppose \( \lim_{x \to 4} f(x) = -6 \) and \( \lim_{x \to 4} h(x) = 8 \). Find \( \lim_{x \to 4} \frac{(f(x) + 3)^2}{h(x) + x} \).

\[
\lim_{x \to 4} \frac{(f(x) + 3)^2}{h(x) + x} = \frac{\lim_{x \to 4} (f(x) + 3)^2}{\lim_{x \to 4} (h(x) + x)} = \frac{\left(\lim_{x \to 4} [f(x) + 3]\right)^2}{\left(\lim_{x \to 4} h(x)\right) + \left(\lim_{x \to 4} x\right)} = \frac{(-6 + 3)^2}{8 + 4} = \frac{3}{4}
\]

3. Graph the function \( f(x) = \begin{cases} 
2x - 6 & x \leq 2 \\
\frac{1}{2}x + 1 & 2 < x \leq 6 \\
-(x - 6)^2 + 4 & 6 < x 
\end{cases} \)
4. Using the function from Problem 3, find
   
   a.) \[ \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 2x - 6 = 2(2) - 6 = -2 \]
   
   b.) \[ \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{1}{2} x + 1 = \frac{1}{2}(2) + 1 = 2 \]
   
   c.) \[ \lim_{x \to 2} f(x) \text{ does not exist because the left and right limits are not equal.} \]

5. Using the function from Problem 3, find
   
   a.) \[ \lim_{x \to 6^-} f(x) = \lim_{x \to 6^-} \frac{1}{2} x + 1 = \frac{1}{2}(6) + 1 = 4 \]
   
   b.) \[ \lim_{x \to 6^+} f(x) = \lim_{x \to 6^+} -(x - 6)^2 + 4 = -(6 - 6)^2 + 4 = 4 \]
   
   c.) \[ \lim_{x \to 6} f(x) = 4 \]

6. Compute \[ \lim_{x \to 4} \frac{x^2 + x - 20}{x - 4} \].
   
   \[ \lim_{x \to 4} \frac{x^2 + x - 20}{x - 4} = \lim_{x \to 4} \frac{(x - 4)(x + 5)}{x - 4} = \lim_{x \to 4} x + 5 = 9 \]

7. Determine \[ \lim_{x \to 13} \frac{x - 13}{x^2 - 169} \].
   
   \[ \lim_{x \to 13} \frac{x - 13}{x^2 - 169} = \lim_{x \to 13} \frac{x - 13}{(x - 13)(x + 13)} = \lim_{x \to 13} \frac{1}{x + 13} = \frac{1}{26} \]

8. Evaluate \[ \lim_{x \to 3^-} \frac{|10x - 30|}{4(5x - 15)} \].
   
   First, observe that when \( x < 3 \), \( x - 3 < 0 \) and \( |x - 3| = -(x - 3) \). Thus, \( \frac{|x - 3|}{x - 3} = -1 \) when \( x < 3 \).
   
   Thus, \[ \lim_{x \to 3^-} \frac{|10x - 30|}{4(5x - 15)} = \lim_{x \to 3^-} \frac{10|x - 3|}{20(x - 3)} = \frac{10}{20}(-1) = -\frac{1}{2} \].

9. Compute \[ \lim_{s \to +\infty} \frac{s^5 + s^4 + s^3 + s^2 + s}{1000s^3} \].
   
   Since \( s \) is going to \( \infty \), we need only consider the ratio of the terms with highest degree. So,
   
   \[ \lim_{s \to +\infty} \frac{s^5}{1000s^3} = \lim_{s \to +\infty} \frac{s^5}{1000s^3} = \lim_{s \to +\infty} \frac{s^2}{1000} = \infty \].
10. Determine \( \lim_{t \to +\infty} \frac{(3t + 7)(4t + 6)(t^2 + 2t + 1)}{7(5t + 1)(t^2 + 12t)(t + 160)}. \)

Since \( t \) is going to \( \infty \), we need only consider the ratio of terms with the highest degree. Instead of multiplying out the numerator and the denominator, we merely go through and pick out the highest order term of each factor. So, we have \( \lim_{t \to +\infty} \frac{(3t + 7)(4t + 6)(t^2 + 2t + 1)}{7(5t + 1)(t^2 + 12t)(t + 160)} = \lim_{t \to +\infty} \frac{(3t)(4t)(t^2)}{7(5t)(t^2)(t)} = \lim_{t \to +\infty} \frac{12t^4}{35t^4} = \frac{12}{35}. \)

11. Evaluate \( \lim_{x \to -\infty} \frac{2x^5}{(x^3 + 2x^2 + 16x + 7)^2}. \)

Since \( x \) is going to \( -\infty \), we need only consider the ratio of terms with the highest degree. Then, \( \lim_{x \to -\infty} \frac{2x^5}{(x^3 + 2x^2 + 16x + 7)^2} = \lim_{x \to -\infty} \frac{2x^5}{(x)^2} = \lim_{x \to -\infty} \frac{2x^5}{x^6} = \lim_{x \to -\infty} \frac{2}{x} = 0. \)

Use the following graph on problems 12 and 13.

12. a.) Is the function \( f(x) \) continuous at \( x = -1 \)?

No. The limit of the function at \( x = -1 \) is 1, but \( f(-1) = 3 \).

b.) Is the function \( f(x) \) continuous at \( x = 0 \)?

Yes. The limit of the function at \( x = 0 \) is equal to \( f(0) = 0 \).

c.) Is the function \( f(x) \) continuous at \( x = 2 \)?

Yes. The limit of the function at \( x = 2 \) is equal to \( f(2) = 4 \).
13. a.) Is the function $f(x)$ differentiable at $x = -1$?

**No. The function is not continuous at $x = -1$, and thus is not differentiable at $x = -1$.**

b.) Is the function $f(x)$ differentiable at $x = 0$?

**Yes. The derivative of at $x = 0$ is 0.**

c.) Is the function $f(x)$ differentiable at $x = 2$?

**No. The graph of the function has a sharp point at $x = 2$.**

14. Consider the function $f(x) = \begin{cases} 
2\sqrt{x} + 3 & x \leq 4 \\
\frac{1}{2}x^2 + B & 4 < x
\end{cases}$

Find a value for $B$ such that $f(x)$ is continuous at $x = 4$.

We want to choose $B$ so that $\lim_{x \to 4^-} f(x) = f(4)$. Since $f(4) = 7$, we need $\lim_{x \to 4^+} f(x) = 7$, so we need the sided limits at 4 to also be 7.

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} 2\sqrt{x} + 3 = 7$, so the left limit works out, and we don’t need to do anything to it.

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \frac{1}{2}x^2 + B = 8 + B$, and we need this limit to be 7. So, we need a value of $B$ so that $8 + B = 7$, i.e. $B = -1$.

Then, when $B = -1$, the sided limits (and therefore the overall limit) will equal 7 at $x = 4$.

15. Consider the function $g(x) = \begin{cases} 
3x^2 + 4 & x \leq 1 \\
mx + b & 1 < x
\end{cases}$

Find values for $m$ and $b$ such that $g(x)$ is differentiable at $x = 1$.

In order for $g(x)$ to be differentiable at $x = 1$, we need the slopes on either side of 1 to be equal, thus we consider the sided limits of $g'(x)$.

$\lim_{x \to 1^-} g'(x) = \lim_{x \to 1^-} 6x = 6$, and $\lim_{x \to 1^+} g'(x) = \lim_{x \to 1^+} m = m$. So, $m = 6$ in order for the slopes to agree and for $g(x)$ to be differentiable at 1.

Now that we know $m = 6$, we need to find a value of $b$ so that $g(x)$ is continuous at $x = 1$. So, we consider the sided limits of $g(x)$.

$\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} 3x^2 + 4 = 7$, and $\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} mx + b = \lim_{x \to 1^+} 6x + b = 6 + b$.

These limits will agree only when $7 = 6 + b$, that is, when $b = 1$. Now, $b = 1$ implies that $\lim_{x \to 1} g(x) = 7$. Since $g(1) = 7$, we have $\lim_{x \to 1} g(x) = g(1)$, and $g(x)$ is continuous at 1.