All problems are worth 2 points each.

1. The product of two positive real numbers $x$ and $y$ is 32.
   (a) Find the minimal sum of the two numbers.
   (b) Find the minimal value of the expression $4x + 2y$.

2. Gerald wants to fence in a rectangular area with two horizontal lengths and five vertical
   lengths. He has a total of 2800 feet of fencing. What is the largest possible rectangular
   area that Gerald can enclose given these specifications?

3. Find the dimensions of the largest rectangle which has one side on the positive $x$-axis,
   one side on the positive $y$-axis, one vertex at the origin, and the opposite vertex on
   the curve $y = 4e^{-x}$. (To get started, you may find it helpful to draw a picture.)

4. Find the point on the curve of $y = \sqrt{x + 1}$ that is closest to the point $(2, 0)$.
   (You may find it helpful to find a formula for the distance between $(2, 0)$ and a general
   point $(x, y)$ on the curve $y = \sqrt{x + 1}$.)

5. A spherical balloon is being filled at a rate of 100 cubic centimeters per minute.
   (a) How fast is the radius increasing when the radius is 50 centimeters?
   (b) How fast is the surface area increasing when the radius is 50 centimeters?
   (Recall that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ and the surface area of a
   sphere is $S = 4\pi r^2$).
6. Suppose that the height of a certain right triangle is always twice the base. Suppose the base is expanding at a rate of 5 inches per second.

(a) How fast is the height of the triangle changing?
(b) How fast is the hypotenuse of the triangle changing?
(c) How fast is the area of the triangle increasing when the base is 10 inches long?

7. Suppose that two trains leave a station at the same time. One train is traveling west at 15 miles per hour and the other is traveling south at 20 miles per hour.

(a) How fast are the trains traveling away from each other when the westbound train is 30 miles from the station?
(b) How fast are the trains traveling away from each other when the southbound train is 60 miles from the station?

8. Suppose a certain rectangular pool is 10 feet long by 20 feet wide by 6 feet high.

(a) Suppose the pool is being filled with water at a rate of 1.5 cubic feet per minute. How fast is the height level of the pool when the pool is half full?
(b) The next day, the water is evaporating at a constant rate. Suppose the height of the water is decreasing by 0.5 feet per day. How fast is the volume of water in the pool decreasing?