Suppose that the following augmented matrix in row-reduced form is equivalent to the augmented matrix of a system of linear equations.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 3 \\
0 & 1 & 1 & 0 & -1 \\
0 & 0 & 0 & 1 & 2 \\
\end{bmatrix}
\]

a.) How many solutions does this system have? How do you know?

The corresponding system of linear equations has infinitely many solutions because there are 4 columns to the left of the vertical bar (which correspond to 4 different variables in the system) but there are only 3 pivots as indicated in red. Therefore, our system is underdetermined.

b.) If the system has exactly one solution, what is it? If the system has infinitely many solutions, give three of them. (Hint: Try writing the above matrix as a system of linear equations.)

If we convert the augmented matrix above back into a system of linear equations, we obtain the following system:

\[
\begin{align*}
w &= 3 \\
x + y &= -1 \\
z &= 2
\end{align*}
\]

Three solutions to this system are (3, 1, -2, 2), (3, 0, -1, 2), and (3, -1, 0, 2).