A company has 3210 employees. Each employee is to be given an ID number that consists of one letter followed by one digit.

a.) How many different ID numbers are possible?

We have two choices to make: (1) choose a letter, and (2) choose a digit. There are 26 ways to choose a letter, and there are 10 ways to choose a digit (don’t forget that zero is an option!). Since the choices are independent, there must be $26 \cdot 10 = 210$ ways to choose a letter and a digit. Thus, there are 210 possible ID numbers using this scheme.

b.) Is it possible to give each employee a different ID number using this scheme?

The company has 3210 employees but only 210 possible ID number, so there are clearly no enough to go around.

c.) Is it possible to give each employee a different ID number if we are allowed one letter and two digits?

Similarly to part (a), we need to choose one letter, one digit, and a second digit. There are 26 ways to choose a letter, and 10 ways to choose the first digit. Since the choice of the second digit is independent of the first digit, there are also 10 ways to choose the second digit. In total, there are $26 \cdot 10 \cdot 10 = 2600$ possible ID numbers with a single letter and two digits. Again, $3210 > 2600$, so there aren’t enough ID numbers to go around.

c*.) Is it possible to give each employee a different ID number if we are allowed two letters and one digit?

Similarly to part (a), we need to choose one letter, a second letter, and one digit. There are 26 ways to choose the first letter, and 10 ways to choose the digit. Since the choice of the second letter is independent of the first letter, there are also 26 ways to choose the second letter. In total, there are $26 \cdot 26 \cdot 10 = 6760$ possible ID numbers with two letters and a single digit. Since $6760 > 3210$, there are enough ID numbers to go around.