Solutions

Consider the matrix \( A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \).

1. Recall that the \((i,j)\)-cofactor of \( A \) (denoted \( C_{i,j} \)) is equal to \((-1)^{i+j} \det A_{i,j}\), where \( A_{i,j} \) denotes the \((i,j)\)-minor of \( A \). Calculate the missing cofactors.

\[
\begin{array}{c|c|c}
(-1)^{1+1} \begin{vmatrix} 3 & 0 \\ 0 & -2 \end{vmatrix} &= 1 \begin{vmatrix} -6 \end{vmatrix} \\
C_{1,1} &= -6 \\
C_{1,2} &= 0 \\
C_{1,3} &= 0 \\
(-1)^{2+1} \begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} &= -(-4) \\
C_{2,1} &= -4 \\
C_{2,2} &= -2 \\
C_{2,3} &= \odot \\
(-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} &= -(0) \\
C_{3,1} &= -12 \\
C_{3,2} &= 0 \\
C_{3,3} &= 3 \\
\end{array}
\]

2. Calculate \( \det A \). (Hint: Observe that \( A \) is a triangular matrix.)

Since \( A \) is triangular, \( \det A \) is the product of diagonal entries, namely \( \det A = 1 \cdot 3 \cdot (-2) = -6 \).

3. If \( A \) is an invertible \( 3 \times 3 \) matrix, then it can be shown that \( A^{-1} = \frac{1}{\det A} \begin{bmatrix} C_{1,1} & C_{2,1} & C_{3,1} \\ C_{1,2} & C_{2,2} & C_{3,2} \\ C_{1,3} & C_{2,3} & C_{3,3} \end{bmatrix} \).

Given the specific matrix \( A \) from above and your answers to parts (1) and (2), find \( A^{-1} \).

Pay close attention to the order of the subscripts in the formula above.

\[
A^{-1} = \frac{1}{-6} \begin{bmatrix} -6 & 4 & -12 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2/3 & 2 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}
\]