Calculus Assessment

Find a solution for each of the following, but be sure to justify your answers! This assignment is due at the beginning of the afternoon class on August 3th.

1. Find the following limits:
   a.) \( \lim_{x \to 1} \cos \left( \frac{4\pi}{x} \right) \)
   b.) \( \lim_{x \to 0} \cos \left( \frac{4\pi}{x} \right) \)
   c.) \( \lim_{x \to \infty} \frac{1 - e^x}{1 + 2e^x} \)
   d.) \( \lim_{x \to 0} x^4 \cos \left( \frac{4\pi}{x} \right) \)

2. Evaluate the following integrals:
   a.) \( \int_1^2 \frac{\ln(x)}{x^2} \, dx \)
   b.) \( \int t^2e^t \, dt \)

3. Suppose you are given a function \( f \) such that \( f(2) = 37 \), \( f(3) = 20 \), \( f'(2) = -0.4 \), and \( f'(3) = 2 \). Approximate \( f(2.001) \).

4. Determine whether each of the following sequences converges or diverges. If it converges, find its limit.
   a.) \( a_n = 1 - (0.2)^n \)
   b.) \( b_n = \frac{n!}{2^n} \)

5. Determine which of the following series are convergent. For a convergent series, find its sum. Otherwise, show that it is divergent.
   a.) \( \sum_{n=1}^{\infty} \frac{1}{2n} \)
   b.) \( \sum_{n=0}^{\infty} (-1)^n \)
   c.) \( \sum_{n=1}^{\infty} \frac{1}{2^n} \)
   d.) \( \sum_{n=0}^{\infty} 0 \)
6. Determine whether each of the series is convergent or divergent.

a.) \( \sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3} \)  
    b.) \( \sum_{n=0}^{\infty} \frac{n^2 + 2}{(n + 1)^3} \)  
    c.) \( \sum_{n=1}^{\infty} \frac{e^n}{n^2} \)  
    d.) \( \sum_{n=0}^{\infty} \frac{3^n n^2}{n!} \)

7. Find a power series representation for \( f(x) = \ln(1 + x) \).

8. Recall that a number \( r \) is rational if there exist integers \( p \) and \( q \) such that \( r = \frac{p}{q} \). A number is irrational if it is not rational. Prove that if \( x \) is a rational number and if \( y \) is an irrational number, then \( x + y \) is irrational.