Bases, Eigenvalues, and Eigenvectors Homework 11
As always, be sure to justify your solutions.

1. Find a basis for each of these subspaces of \( \mathbb{R}^n \).
   (a) The null space of \( A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \).
   (b) All vectors whose components are equal in \( \mathbb{R}^4 \).
   (c) All vectors whose components add up to zero \( \mathbb{R}^4 \).
   (d) All vectors that are perpendicular to \((1, 1, 0, 0)\) and \((1, 0, 1, 1)\).

2. Find three different bases for the column space of \( A = \begin{bmatrix} 2 & 5 & -8 & 7 \\ -1 & 5 & 4 & 7 \\ 0 & 5 & 0 & 7 \end{bmatrix} \).

3. Find three different bases for the null space of \( A = \begin{bmatrix} 2 & 5 & -8 & 7 \\ -1 & 5 & 4 & 7 \\ 0 & 5 & 0 & 7 \end{bmatrix} \).

4. Suppose \( S \) is a 5-dimensional subspace of \( \mathbb{R}^6 \). Prove or disprove:
   (a) Every basis for \( S \) can be extended to a basis for \( \mathbb{R}^6 \) by adding one more vector.
   (b) Every basis for \( \mathbb{R}^6 \) can be reduced to a basis for \( S \) by removing one vector.