Diagonalization Homework 12
As always, be sure to justify your solutions.

1. Find the eigenvalues of
   \[ B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \]

2. Prove that the eigenvalues of \( A \) are the same as the eigenvalues of \( A^T \) for any square matrix \( A \).

3. Construct any 3 \( \times \) 3 matrix \( M \) with positive entries so that the sum of the each column is equal to 1. If \( \mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \), prove \( \mathbf{e} \) is an eigenvector of \( M^T \). From above, this means \( \lambda = 1 \) is also an eigenvalue of \( M \). Find an eigenvector of \( M \) corresponding to \( \lambda = 1 \).

4. Let \( A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \). Diagonalize \( A \). Find a formula for \( A^k \).

5. You found the eigenvalues of
   \[ B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \]

   above. Can you diagonalize \( B \)? If so, diagonalize \( B \). If not, explain why not.

6. Let \( A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix} \). Diagonalize \( A \). Find a formula for \( A^k \).