Diagonalization Homework 13
As always, be sure to justify your solutions.

1. Let \( A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix} \). Diagonalize \( A \). Find a formula for \( A^k \). (Hint: The eigenvalues for \( A \) are 2 and 8.)

2. Let \( A = \begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix} \). Diagonalize \( A \). (Hint: One eigenvalue is 5 and one eigenvector is \( \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \).)

3. Construct a nondiagonal \( 2 \times 2 \) matrix that is diagonalizable but not invertible.

4. Construct a nonzero \( 2 \times 2 \) matrix that is invertible but not diagonalizable.

5. You found a formula for \( A^k \) on this morning’s homework, where \( A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \). For an \( n \times n \) matrix \( M \), define

\[
e^M = \sum_{n=0}^{\infty} \frac{M^n}{n!}\]

Find a formula for \( e^A \), where \( A \) is as above.

6. You diagonalized

\[
A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\]

on this morning’s homework. Find a formula for \( A^k \). Use this to find a formula for \( e^A \).