Practice test:

MA162-020
2010-06-16

1. The citizens of a warm country desire a new temperature scale so that they would see the zero degree temperature sometimes and their high temperatures would not appear too high. The proposed new scale Newtemp, or N for short, has the conversion formula: $N = \frac{7}{5}(F - 45)$ where F is the Fahrenheit temperature.

(a) When is the Fahrenheit temperature equal to 3 times the New temperature?

(b) Can 7 times the Fahrenheit temperature be equal to 24 more than 5 times the New temperature? ($7F = 5N + 24$) Why or why not?

2. A courier travels from the city Alton with coordinates (0, 0) to the city Crawford with coordinates (120, 130). He must pass through exactly one of the cities Brady with coordinates (70, 65) or Dalton (65, 70) along the way. Assume he travels straight lines between cities.

(a) Which city should he pass through (Brady or Dalton) in order to minimize his trip distance from Alton to Crawford?

(b) What is the total minimum length of his trip from Alton to Crawford?

3. Point A has coordinates (5, 3), and point B has coordinates (0, 8).

a) What is the distance from A to B and what is the slope of the line joining A to B?

b) Find the number y so that the point C with coordinates (9, y) lies in the first quadrant and triangle ABC is a right triangle with right angle at B.

4. The Rightest company manufactures testers for electrical circuits. The cost function for their manufacturing line is $C = 5x + 7800$, where x is the number of testers produced per month and C is measured in dollars. The testers generate a revenue of \$8 per unit.

a) Determine the linear profit function for the Rightest company in the usual form: $P = mx + b$, assuming they can sell all the testers they manufacture.

b) Determine the break-even production x and the break-even cost C at the break-even production.

5. In a free market, the supply equation for a supplier of soybean is $x = 20p + 100$ where the price p is in dollars and x is in bushels. When the price is \$2 per bushel the demand is 480 bushels. When the price goes up to \$12 per bushel the demand drops to 0 bushels. Assuming that the demand function is also linear, find the equilibrium price and the number of bushels supplied at that equilibrium price.

Here are some brief solutions:

1. It is nice to clear fractions first: $N = \frac{7}{5}(F - 45)$ means $5N = 7F - 315$

1a. $F = 3N$ and $5N = 7F - 315$, so $5N = 7(3N) - 315$ and $5N = 21N - 315$ so $16N = 315$ and $N = 315/16 = 19.6875$ and $F = 3N = 945/16 = 59.0625$.

1b. If $7F = 5N + 24$, then $5N = 7F - 24$, but we already know $5N = 7F - 315$ and there is no solution to $7F - 24 = 7F - 315$, since $-24 \neq -315$.

2. Just calculate each of the distances:

$AB = \sqrt{70^2 + 65^2} = \sqrt{4900 + 4225} = \sqrt{9125} \approx 95.52$

$BC = \sqrt{(120 - 70)^2 + (130 - 65^2)} = \sqrt{50^2 + 65^2} = \sqrt{6725} \approx 82.00$

$AD = \sqrt{65^2 + 70^2} = \sqrt{9125} \approx 95.52$

$DC = \sqrt{(120 - 65)^2 + (130 - 70)^2} = \sqrt{55^2 + 60^2} = \sqrt{6625} \approx 81.39$

2a. Dalton is shorter.

2b. The total length is $ADC = \sqrt{9125} + \sqrt{6625} \approx 176.92$

If he skipped Brady and Dalton, then he would take $AC = \sqrt{120^2 + 130^2} = \sqrt{31300} = 176.92$, so Dalton is basically on the way. Be careful that this problem never asks for AC, nor does it ask him to visit both Brady and Dalton.

3a. $AB = \sqrt{(5 - 0)^2 + (3 - 8)^2} = \sqrt{5^2 + 5^2} = \sqrt{50} \approx 7.07$ and $m_{AB} = \frac{3-8}{5-0} = \frac{-5}{5} = -1$.

3b. $m_{BC} = +1$ and so the equation of the line BC is $y - 8 = 1(x - 0)$, that is, $y = x + 8$. Since $x = 9$, $y = 17$.

4a. $R = 8x$ so $P = R - C = 8x - (5x + 7800) = 3x - 7800$.

4b. $P = 0$ means $3x = 7800$ (profit per unit just covers the rent), so $x = \frac{7800}{3} = 2600$ produced at a cost of $5(2600) + 7800 = \$20,800$ or more simply, costing as much as we made, $8(2600) = \$20,800$.

5. The demand equation is $d - 480 = \frac{480-0}{2-12}(p - 2)$, but this can be simplified to $d = 576 - 48p$. In equilibrium, the amount supplied, x, is equal to the amount demanded, d, so we set $20p + 100 = 576 - 48p$ and solve for p in the usual way: $68p = 476$, $p = 7$ so the equilibrium price is \$7, and the equilibrium production/demand is $576 - 48(7) = 240$ or $20(7) + 100 = 240$, that is, 240 bushels.