Name: ______________________________

MA162-020
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Quiz on 2.1: Systems of equations

Solve the system of equations:

$$\begin{array}{rcrcr} y & = & 2x & + & 1 \\ x & + & y & = & 4 \end{array}$$

Solve the system of equations:

$$\begin{array}{rcrcr} 28p & - & x & = & -97 \\ 40p & + & x & = & 420 \end{array}$$

Solve the system of equations:

$$\begin{array}{rcrcr} x & & & = & 43 \\ & & y & = & -17 \end{array}$$

Solve the system of equations:

$$\begin{array}{rcr} x & = & 43 \\ x & = & -17 \end{array}$$

Solve the system of equations:

$$\begin{array}{rcrcr} x & + & y & = & 43 \\ x & + & y & = & -17 \end{array}$$

Solve the system of equations:

$$\begin{array}{rcrcr} x & + & y & = & 43 \\ x & - & y & = & -17 \end{array}$$

Solve the system of equations:

$$\begin{array}{rcrcr} x & + & y & = & 1 \end{array}$$

Solve the system of equations:

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 10 \\ & & y & + & 2z & = & 4 \\ & & & & z & = & 1 \end{array}$$

Examples for 2.1: systems of equations

Chapter 1 discussed the intersection of two lines. Real business decisions usually involve more than one or two factors, and so lines in the plane no longer suffice. Maybe your company actually produces **two** products, and you need to calculate the profit:

$$P = 3x + 4y$$

Maybe you just know that you made \$10 profit when $x = 2$ and $y = 1$, and \$30 when $x = 6$ and $y = 3$. How much profit do you make at $x = 4$ and $y = 2$? Well $x = 4$ is halfway between $x = 2$ and $x = 6$, and $y = 2$ is halfway between $y = 1$ and $y = 3$, so surely the profit is halfway between \$10 and \$30: $P = \$20$. We can work on one variable at a time to figure out the situation.

However, the more variables, the harder it is to graph, and the easier it is to get confused. We have to get better at solving equations. The old methods are too slow and require way too much writing. We also need to be able to recognize badly phrased problems earlier so we don't waste time on problems without solutions.

All of the problems in this chapter are based on the idea of finding solutions, like $(x = 4, y = 2, P = 20)$, that satisfy **multiple equations simulatenously**, like:

$$\begin{array}{rcrcrcr} -3x & - & 4y & + & P & = & 0 \\ x & + & y & & & = & 6 \\ 2x & + & y & + & P & = & 30 \end{array}$$

We need to decide **how many solutions** it has (the answer in this class will always be **none**, **one**, or **infinitely many**), and how to describe all of the solutions. When there are none, this is easy: **NO SOLUTION**

when there is one, this is standard, we just say what it is: $(x = 4, y = 2, P = 20)$

when there are infinitely many we need to be careful to give a **useful** answer, not just "too many to write down dude" or "here is one of them, but I guess there are more." We divide the variables up into **free variables** and dependent variables. The free variables can take any value their little old heart desires, but then the dependent variables are forced to take on exactly one value to compensate. For instance in the system:

$$-2x + y = 1$$

There are ton of solutions, $(x = 0, y = 1)$ and $(x = 1, y = 3)$ are two of them, but there are more. On way to write down **all** solutions is to say "x can be anything, but whatever you choose for x, you have to choose $y = 2x + 1$". In other words the solutions form a line. We write this answer as:

$$\begin{array}{rcl} x & = & \text{free} \\ y & = & 2x + 1 \end{array}$$

Sometimes there are no solutions. For instance the system:

$$\begin{array}{rcl} x & = & 0 \\ x & = & 1 \end{array}$$

has no solution. If $x = 0$, then x cannot be 1, so we can satisfy the first but not the second. If $x = 1$, then x cannot be 0, so we can satisfy the second, but not the first. There is no way to satisfy both. There is **no solution**.