Name: ______________________________

Quiz on 2.2: Systematic elimination of variables

1. Write the system of equations as an augmented matrix (do not solve):

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & 4 \\ x & - & 2y & - & 3z & = & 4 \\ 7x & + & 6y & - & 5z & = & 8 \end{array}$$

2. Write the augmented matrix as a system of equations (do not solve):

$$\left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 4 & 3 & 2 & 1 \\ 8 & -7 & -6 & -5 \\ 0 & 1 & 0 & 9 \end{array}\right)$$

3. Circle the pivots. Use one row operation to make a new zero below the first pivot, and label the row operation.

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 9 & 10 & 11 \end{array}\right) \longrightarrow$$

4. Use elimination to solve the following system:

$$\left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 2 & 0 & -3 & -7 \\ 2 & 1 & -3 & -5 \\ -2 & -1 & 6 & 14 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right) \longrightarrow$$

$$\left(\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right) \longrightarrow$$

$$\left(\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right) \longrightarrow$$

$$\left(\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right)$$

Solution: ($x =$ ____________, $y =$ ____________, $z =$ ____________)

Examples for 2.2: Systematic elimination of variables

Our systematic elimination of variables use one main operation on systems of equations and occasionally two auxillary operations. The main operation is to take a multiple of one equation and add it to another. For instance, in the following we change the second equation by subtracting 4 times the first equation from it.

$$\begin{array}{rcrcr} x & + & 2y & = & 3 \\ 4x & + & 5y & = & 6 \end{array} \xrightarrow{\text{2ND}-4\times\text{1ST}} \begin{array}{rcrcr} x & + & 2y & = & 3 \\ 0 & - & 3y & = & -6 \end{array}$$

Both pairs of equations have exactly the same solutions, but the right-hand pair is much easier to solve: $-3y = -6$ just means $y = 2$ and then $x + 2(2) = 3$ means $x = -1$. We can write this in terms of the equations using the first auxillary operation: multiplying an equation by a non-zero number.

$$\begin{array}{rcrcr} x & + & 2y & = & 3 \\ 0 & - & 3y & = & -6 \end{array} \xrightarrow{(-1/3)\times\text{2ND}} \begin{array}{rcrcr} x & + & 2y & = & 3 \\ 0 & + & y & = & 2 \end{array} \xrightarrow{\text{1ST}-2\times\text{2ND}} \begin{array}{rcrcr} x & + & 0 & = & -1 \\ 0 & + & y & = & 2 \end{array}$$

We waste time writing down the variables and the + and = signs in the middle steps. It is faster to label columns and just write down the coefficients in an **augmented matrix**:

$$\left(\begin{array}{cc|c} x & y & \text{RHS} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right) \xrightarrow{R_2-4R_1} \left(\begin{array}{cc|c} x & y & \text{RHS} \\ 1 & 2 & 3 \\ 0 & -3 & -6 \end{array}\right) \xrightarrow{(-1/3)R_2} \left(\begin{array}{cc|c} x & y & \text{RHS} \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{array}\right) \xrightarrow{R_1-2R_2} \left(\begin{array}{cc|c} x & y & \text{RHS} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right)$$

A bigger example:

$$\begin{array}{rcrcrcr} x & + & 3y & + & 2z & = & 13 \\ x & + & 4y & + & 3z & = & 18 \\ 3x & + & 7y & + & 5z & = & 32 \end{array} \xrightarrow{\text{as matrix}} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ 1 & 3 & 2 & 13 \\ 1 & 4 & 3 & 18 \\ 3 & 7 & 5 & 32 \end{array}\right) \xrightarrow[R_3-3R_1]{R_2-R_1} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ 1 & 3 & 2 & 13 \\ \mathbf{0} & 1 & 1 & 5 \\ \mathbf{0} & -2 & -1 & -7 \end{array}\right)$$

$$\xrightarrow{R_3+2R_1} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ 1 & 3 & 2 & 13 \\ 0 & 1 & 1 & 5 \\ 0 & \mathbf{0} & 1 & 3 \end{array}\right) \xrightarrow[R_1-2R_3]{R_2-R_3} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ 1 & 3 & \mathbf{0} & 7 \\ 0 & 1 & \mathbf{0} & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_1-3R_2} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ 1 & \mathbf{0} & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Elimination proceeds in two stages: in the first stage, you move left to right, making zeros below. In the second stage, you move right to left, making zeros above. In each row, the first nonzero entry is called the **pivot**. In stage one, the goal is to make sure no column has two pivots; when you succeed this is called **REF**. In stage two, the goal is to make sure a column with a pivot in it (a "pivot column") has exactly one 1, and the rest of its entries are 0; when you succeed this is called **RREF**.

In stage one, you repeat a single idea until you are in REF: (1) circle all the pivots, (2) in the left-most column that has at least two pivots, the top pivot is **activated**, and the other pivots in that column are **targets**, (3) use the following row operation to have the activated agent eliminate the target:

$$R_{\text{target row}} - \frac{\text{target entry}}{\text{active pivot}} R_{\text{active row}}$$

Stage two is similar, but one chooses targets that are right-most nonzero entries above a pivot, and one uses $\frac{1}{\text{active pivot}} R_{\text{active row}}$ to make the pivot 1.