Name: ______________________________

MA162-020
2010-06-22

Quiz on 2.3: Over and under determined systems

1. Write the system of equations as an augmented matrix. What are the solutions?

$$\begin{array}{ccccccc} x & + & 0 & + & 3z & = & 4 \\ & & y & - & 3z & = & 4 \end{array}$$

2. Write the augmented matrix as a system of equations. What are the solutions?

$$\left(\begin{array}{cc|c} x & y & \text{RHS} \\ \hline 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{array}\right)$$

3. For what value of k does the following system have no solution?

$$\begin{array}{ccccccc} x & + & 0 & + & 0 & = & 1 \\ & & y & + & 0 & = & 2 \\ & & & & kz & = & 3 \end{array}$$

4. For what value of k does the following system have no solution?

$$\left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & k & 6 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} \quad & \quad & \quad & \quad \\ & & & \\ & & & \end{array}\right)$$

5. For what value of k does the following system have no solution?

$$\left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 1 & 2 & 3 & 4 \\ 0 & 5 & k & 6 \\ 0 & 7 & 8 & 9 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} \quad & \quad & \quad & \quad \\ & & & \\ & & & \end{array}\right)$$

Examples for 2.3: Over and under determined systems

After reducing to REF, each column has **at most one** pivot. Columns with a pivot are called **pivot columns** and the variable associated to that column is called a **pivot variable**. Columns without pivots are called **free columns**, and the associated variable is called a **free variable**. When we reduce the matrix to RREF, pivot will be one, and the only other non-zero entries will be free variables that are further to the right and the RHS column. This allows us to express pivot variables as functions of the free variables. For example:

$$\left(\begin{array}{ccccc|c} x & y & z & s & t & \text{RHS} \\ \hline 1 & 2 & 3 & 0 & 4 & 5 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{\text{as system}} \begin{aligned} x+2y+3z+4t &= 5 \\ s+6t &= 7 \\ 0 &= 0 \end{aligned} \xrightarrow{\text{solve}} \begin{aligned} x &= 5-2y-3z-4t \\ s &= 7-6t \\ y,z,t &= \text{free} \end{aligned}$$

The last equation is a bit silly, $0 = 0$. However, this will actually happen in real problems: the same requirement can be stated in two different sounding but equivalent ways. However, in some real problems, two requirements can actually conflict. Instead of $0 = 0$, sometimes you end up with $0 = 13$. No matter what magic you work, no value of the variables will make $0 = 13$, and there will be no solution:

$$\left(\begin{array}{ccccc|c} x & y & z & s & t & \text{RHS} \\ \hline 1 & 2 & 3 & 0 & 4 & 5 \\ 0 & 0 & 0 & 1 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 13 \end{array}\right) \xrightarrow{\text{as system}} \begin{aligned} x+2y+3z+4t &= 5 \\ s+6t &= 7 \\ 0 &= 13 \end{aligned} \xrightarrow{\text{solve}} \textbf{No solution}$$

Another example:

$$\left(\begin{array}{cccc|c} x & y & z & t & \text{RHS} \\ \hline 2 & -1 & -4 & 1 & -21 \\ 4 & -1 & -5 & 2 & -19 \\ 2 & -1 & -2 & 3 & -9 \end{array}\right) \xrightarrow[R_3-2R_1]{R_2-R_1} \left(\begin{array}{cccc|c} x & y & z & t & \text{RHS} \\ \hline 2 & -1 & -4 & 1 & -21 \\ 0 & 1 & 3 & 0 & 23 \\ 0 & 0 & 2 & 2 & 12 \end{array}\right) \xrightarrow{\frac{1}{2}R_3} \left(\begin{array}{cccc|c} x & y & z & t & \text{RHS} \\ \hline 2 & -1 & -4 & 1 & -21 \\ 0 & 1 & 3 & 0 & 23 \\ 0 & 0 & 1 & 1 & 6 \end{array}\right)$$

$$\xrightarrow[R_2-3R_3]{R_1+4R_3} \left(\begin{array}{cccc|c} x & y & z & t & \text{RHS} \\ \hline 2 & -1 & 0 & 5 & 3 \\ 0 & 1 & 0 & -3 & 5 \\ 0 & 0 & 1 & 1 & 6 \end{array}\right) \xrightarrow{R_1+R_2} \left(\begin{array}{cccc|c} x & y & z & t & \text{RHS} \\ \hline 2 & 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & -3 & 5 \\ 0 & 0 & 1 & 1 & 6 \end{array}\right) \xrightarrow{\frac{1}{2}R_1} \left(\begin{array}{cccc|c} x & y & z & t & \text{RHS} \\ \hline 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -3 & 5 \\ 0 & 0 & 1 & 1 & 6 \end{array}\right)$$

$$\xrightarrow{\text{equations}} \begin{cases} x+t=4 \\ y-3t=5 \\ z+t=6 \end{cases} \xrightarrow{\text{solution}} \begin{cases} x=4-t \\ y=5+3t \\ z=6-t \\ t=\text{free} \end{cases}$$