Name: ____________________________

HWA3 quiz

1. Find a value of b so that the following system does not have a unique solution in (t, v, z). For that value of b, indicate how many solutions (t, v, z) the system has.

$$\left\{\begin{array}{rcrcrcr} & & 3v & + & 2z & = & 0 \\ -t & - & 2v & - & 2z & = & -1 \\ -2t & + & bv & + & 3z & = & -2 \end{array}\right\}$$

2. Find the RREF of this matrix:

$$\left(\begin{array}{rrr|r} 1 & -1 & 0 & 3 \\ 3 & -2 & -2 & 4 \\ -2 & 2 & 1 & -4 \end{array}\right)$$

3. We want to invest a total of \$45,000 in two funds A and B that have yields of 6% and 8% interest per year, respectively. We want the total interest received after 1 year to be \$3,300.00. How much should be invested in each fund?

4. A building contractor is planning to build an apartment complex with one, two or three bedroom apartments. Let x,y,z respectively denote the number of apartments of each type to be built. The builder will spend a total of \$4,644,000, and the costs for the three types of apartments are \$17,000, \$28,000, and \$44,000 respectively. He plans to build a total of 168 apartments, and will build as many one bedroom apartments as he builds both two and three bedroom apartments combined. Write down equations that indicate: (1) the total number of apartments built, (2) the total cost, and (3) how he will balance one-bedroom apartments against larger apartments. You may want to solve the system on your own paper.

Examples for HWA3

1. To find the value of a parameter that makes a system have 0 or infinitely many solutions, first convert to a matrix, then to REF. The parameter should appear in a "pivot": if you make that pivot 0, then that should make the system degenerate to 0 or infinitely many solutions. For example:

$$\left\{\begin{array}{ccccc} 2x & + & ky & = & 3 \\ 4x & + & 7y & = & 8 \end{array}\right\} \xrightarrow{\text{as matrix}} \left(\begin{array}{cc|c} x & y & \text{RHS} \\ \hline 2 & k & 3 \\ 4 & 7 & 8 \end{array}\right) \xrightarrow{R_2-2R_1} \left(\begin{array}{cc|c} x & y & \text{RHS} \\ \hline 2 & k & 3 \\ 4-2(2) & 7-2(k) & 8-2(3) \end{array}\right)$$

$$\xrightarrow{\text{simplify}} \left(\begin{array}{cc|c} x & y & \text{RHS} \\ \hline 2 & k & 3 \\ 0 & 7-2k & 2 \end{array}\right) \xrightarrow{\text{no solution when}} \{7-2k=0\} \xrightarrow{\text{solve}} \{k=7/2=3.5\}$$

Infinitely many solutions would be when the bottom-right number was a 0 (rather than a 2), since then $k = 7/2$ would make the last row say that "$0 = 0$".

2. Finding the RREF of a matrix is systematic (easy, once you get the hang of it). For example

$$\left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline -1 & 1 & 3 & -1 \\ 2 & -1 & -4 & 2 \\ 2 & -2 & -6 & 2 \end{array}\right) \xrightarrow[R_3+2R_1]{R_2+2R_1} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline -1 & 1 & 3 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) (\text{REF!}) \xrightarrow{R_1-R_2} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline -1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{-R_1} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) (\text{RREF!}) \xrightarrow{\text{equations}} \left\{\begin{array}{rcl} x-z & = & 1 \\ y+2z & = & 0 \\ 0 & = & 0 \end{array}\right\} \xrightarrow{\text{solve}} \left\{\begin{array}{rcl} x & = & 1+z \\ y & = & -2z \\ z & = & \text{free} \end{array}\right\}$$

3. Let x be the number of thousands of dollars invested in fund A, and y be the number of thousands of dollars invested in fund B. The interest received will be $6\%x + 8\%y$ and the total amount invested is $x + y$. We write down the equations, turn them into a matrix, RREF, and read the solution:

$$\left\{\begin{array}{rcrcr} x & + & y & = & 45 \\ 0.06x & + & 0.08y & = & 3.3 \end{array}\right\} \xrightarrow{\text{matrix}} \left(\begin{array}{cc|c} 1 & 1 & 45 \\ 0.06 & 0.08 & 3.3 \end{array}\right) \xrightarrow{100R_2} \left(\begin{array}{cc|c} 1 & 1 & 45 \\ 6 & 8 & 330 \end{array}\right)$$

$$\xrightarrow{R_2-6R_1} \left(\begin{array}{cc|c} 1 & 1 & 45 \\ 0 & 2 & 60 \end{array}\right) \xrightarrow{\frac{1}{2}R_2} \left(\begin{array}{cc|c} 1 & 1 & 45 \\ 0 & 1 & 30 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{cc|c} 1 & 0 & 15 \\ 0 & 1 & 30 \end{array}\right) \xrightarrow{\text{equations}} \left\{\begin{array}{c} x=15 \\ y=30 \end{array}\right\}$$

So we should invest 15 thousand dollars in fund A, and 30 thousand dollars in fund B.

4. The total number of apartments built is $x + y + z$. The total cost is in thousands of dollars is $17x + 28y + 44z$. The last requirement says the number of one bedrooms, x, is equal to the sum of the number of two and three bedrooms, $y + z$. In other words:

$$\left\{\begin{array}{rcl} x+y+z & = & 168 \\ x & = & y+z \\ 17x+28y+44z & = & 4644 \end{array}\right\} \xrightarrow{\text{rearrange}} \left\{\begin{array}{ccccccc} x & + & y & + & z & = & 168 \\ x & - & y & - & z & = & 0 \\ 17x & + & 28y & + & 44z & = & 4644 \end{array}\right\} \xrightarrow{\text{matrix}} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 1 & 1 & 1 & 168 \\ 1 & -1 & -1 & 0 \\ 17 & 28 & 44 & 4644 \end{array}\right)$$

$$\xrightarrow[R_3-17R_1]{R_2-R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 168 \\ 0 & -2 & -2 & -168 \\ 0 & 11 & 27 & 1788 \end{array}\right) \xrightarrow{\frac{-1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 168 \\ 0 & 1 & 1 & 84 \\ 0 & 11 & 27 & 1788 \end{array}\right) \xrightarrow{R_3-11R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 168 \\ 0 & 1 & 1 & 84 \\ 0 & 0 & 16 & 864 \end{array}\right) \xrightarrow{\frac{1}{16}R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 168 \\ 0 & 1 & 1 & 84 \\ 0 & 0 & 1 & 54 \end{array}\right) \xrightarrow[R_2-R_3]{R_1-R_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 114 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 54 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 84 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 54 \end{array}\right) \xrightarrow{\text{equations}} \left\{\begin{array}{c} x=84 \\ y=30 \\ z=54 \end{array}\right\}$$

So he will make 84 one-bedroom apartments, 30 two bedroom apartments, and 54 three bedroom apartments.