Practice test:

MA162-020
2010-06-24

1. For what value of k is the system

$$\begin{bmatrix} x & - & 2y & + & 5z & = & -8 \\ -2x & + & y & + & 3z & = & 17 \\ & & y & + & kz & = & 0 \end{bmatrix}$$

inconsistent (i.e. has no solution)?

2. Given the system of equations

$$\begin{bmatrix} -x & + & y & - & 3z & = & -1 \\ 2x & - & y & + & 5z & = & 1 \\ x & - & 2y & + & 7z & = & 2 \end{bmatrix}$$

a) Write the augmented matrix for the system.

b) Carry out standard row reductions to convert the augmented matrix to REF (row echelon form). Be sure to describe your reductions in standard notation. Just giving the final form will receive no credit.

3. Here is the augmented matrix of a linear system of equations. As usual, the variables are mentioned for your convenience.

x	y	z	w	RHS
1	0	2	0	4
0	1	3	0	5
0	0	0	1	6

(a) Is this matrix in REF or RREF or neither of these?

(b) Finish the solution process as needed and determine the complete solution of the system by filling in the answers below. If a variable is free, then enter the word "free" as its value. be sure to show all calculations.

4. For the following word problem: (a) Write down variables describing the (numerical) business decision to be made, (b) write down equations that constrain your decision, (c) convert the equations to an augmented matrix. You need not solve the system.

Old Man Willerby does not care about profit. He cares about hard work, sweat, and operating at full capacity. His blouse factory is run at peak efficiency making three types of blouses ("shirts for women"): sleeveless, short sleeve, and long sleeve. Sleeveless blouses require 9 minutes of cutting, 22 minutes of sewing, and 6 minutes of packaging. Short sleeve blouses require 12 minutes of cutting, 24 minutes of sewing, and 8 minutes of packaging. Long sleeve blouses require 15 minutes of cutting, 28 minutes of sewing, and 8 minutes of packaging. Willerby's factory is staffed to provide 80 labor-hours of cutting, 160 labor-hours of sewing, and 48 labor-hours of packaging. If he wants every labor-hour used, what should he set his production goals to be? How many sleeveless, how many short sleeve, and how many long sleeve blouses should he direct his workers to make?

5. TBD. Perhaps "here is a matrix in REF, convert to RREF."

Plausible answers:

1. First of, convert to matrices to you can tell which are variables and which are numbers we don't know yet. We can decide if there are free variables in REF, so lets go pivotting!

$$\left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 1 & -2 & 5 & -8 \\ -2 & 1 & 3 & 17 \\ 0 & 1 & k & 0 \end{array}\right) \xrightarrow{R_2+2R_1} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 1 & -2 & 5 & -8 \\ 0 & -3 & 13 & 1 \\ 0 & 1 & k & 0 \end{array}\right) \xrightarrow{R_3+\frac{1}{3}R_2} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 1 & -2 & 5 & -8 \\ 0 & -3 & 13 & 1 \\ 0 & 0 & k+\frac{13}{3} & \frac{1}{3} \end{array}\right)$$

The only way to have an inconsistent system is if $k + \frac{13}{3} = 0$ so that the last equation would read $0z = \frac{1}{3}$ (and no z is so powerful that $0z$ is bigger than 0). Hence the k we want is the one that makes $k + \frac{13}{3} = 0$,

$$k = -\frac{13}{3}$$

2. The matrix is formed by taking the coefficients (numbers) from the variables, paying attention to order.

$$\left[\begin{array}{rcrcrcr} -x & + & y & - & 3\,z & = & -1 \\ 2\,x & - & y & + & 5\,z & = & 1 \\ x & - & 2\,y & + & 7\,z & = & 2 \end{array}\right] \xrightarrow{\text{to matrix}} \left(\begin{array}{rrr|r} -1 & 1 & -3 & -1 \\ 2 & -1 & 5 & 1 \\ 1 & -2 & 7 & 2 \end{array}\right)$$

$$\xrightarrow[R_2+R_1]{R_2+2R_1} \begin{pmatrix} -1 & 1 & -3 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 4 & 1 \end{pmatrix} \xrightarrow{R_3+R_2} \begin{pmatrix} -1 & 1 & -3 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 3 & 0 \end{pmatrix} \quad \text{REF!}$$

3. Above and below the pivots are 0s, and the pivots themselves are 1s, so the matrix is both in REF and RREF.

$$\left[\begin{array}{cccc|c} x & y & z & w & \text{RHS} \\ \hline 1 & 0 & 2 & 0 & 4 \\ 0 & 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right] \xrightarrow{\text{equations}} \left\{\begin{array}{rcl} x+2z & = & 4 \\ y+3z & = & 5 \\ w & = & 6 \end{array}\right\} \xrightarrow{\text{solve}} \begin{cases} x = 4-2z \\ y = 5-3z \\ z = \text{free} \\ w = 6 \end{cases}$$

4. The business decision is how many sleeveless, short-sleeve, and long-sleeve blouses to make. We name each quantity with a variable:

$$\begin{cases} x = \text{how many sleeveless blouses to make} \\ y = \text{how many short-sleeve blouses to make} \\ z = \text{how many long-sleeve blouses to make} \end{cases}$$

If our answer is $(x = 2, y = 3, z = 4)$ then that means the factory should try to produce 2 sleeveless blouses, 3 short-sleeve blouses, and 4 long-sleeve blouses. That would require $2(9)+3(12)+4(15) = 114$ minutes of cutting, $2(22) + 3(24) + 4(28) = 228$ minutes of sewing, and $2(6) + 3(8) + 4(8) = 68$ minutes of packaging.

We are supposed to use $80(60) = 4800$ minutes of cutting, $160(60) = 9600$ minutes of sewing, and $48(60) = 2880$ minutes of packing, so clearly our answer is too small. We need to write down the requirements as equations:

$$\left\{\begin{array}{rcrcrcrl} 9x & + & 12y & + & 15z & = & 4800 & \text{(cutting time)} \\ 22x & + & 24y & + & 28z & = & 9600 & \text{(sewing time)} \\ 6x & + & 8y & + & 8z & = & 2880 & \text{(packaging time)} \end{array}\right\} \xrightarrow{\text{as matrix}} \left(\begin{array}{ccc|c} x & y & z & \text{RHS} \\ \hline 9 & 12 & 15 & 4800 \\ 22 & 24 & 28 & 9600 \\ 6 & 8 & 8 & 2880 \end{array}\right)$$