

## Quiz 4.1: Pivotting

For each problem below: Circle the new pivot, and perform the row operations needed to make it a basic variable (make it look like a pivot). Answer the question following the table.

1. A lucky pivot: Make  $X$  a basic variable and  $U$  a free variable.

| $X$ | $Y$ | $U$ | $V$ | $W$ | $P$ | RHS |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 1   | 0   | 0   | 0   | 10  |
| 1   | 4   | 0   | 1   | 0   | 0   | 11  |
| 1   | 6   | 0   | 0   | 1   | 0   | 12  |
| -5  | -8  | 0   | 0   | 0   | 1   | 0   |

Can you read the maximum value of  $P$  subject to  $X, Y, U, V, W \geq 0$ ?

2. A bad pivot: make  $X$  basic and  $V$  free.

| $X$ | $Y$ | $U$ | $V$ | $W$ | $P$ | RHS |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 1   | 0   | 0   | 0   | 10  |
| 1   | 4   | 0   | 1   | 0   | 0   | 11  |
| 1   | 6   | 0   | 0   | 1   | 0   | 12  |
| -5  | -8  | 0   | 0   | 0   | 1   | 0   |

What (bad thing) happens if you set the free variables to 0?

3. An unlucky pivot: Make  $Y$  a pivot and  $W$  a free variable.

| $X$ | $Y$ | $U$ | $V$ | $W$ | $P$ | RHS |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 1   | 0   | 0   | 0   | 10  |
| 1   | 4   | 0   | 1   | 0   | 0   | 11  |
| 1   | 6   | 0   | 0   | 1   | 0   | 12  |
| -5  | -8  | 0   | 0   | 0   | 1   | 0   |

Can you read the maximum yet?

## Examples 4.1: Pivot selection

The simplex algorithm works by **swapping** the roles of a **free** variable with a **basic** variable. The old free variable, becomes a new basic variable. The pair of variables to swap is called a **pivot**.

Research into the optimal choice of pivot is on-going. Some million dollar companies have approximately only one product: a pivoting strategy.

There are **two decisions** to make. The first is which (old) free variable is going to become the (new) basic variable. In our optimal solution, the free variables are going to be set to 0. If a free variable contributes profit (if there is a negative number in the bottom row of its column), then we do not want it to be 0! Hence it is a candidate to become basic.

The book suggests a “greedy approach”: choose the basic variable that makes the largest marginal contribution to profit (choose the most negative number in the bottom row). The quiz #3 indicates why this might be a poor strategy. Most textbooks suggest “Bland’s rule”: choose the left-most candidate to become the basic variable. It is simple and works fairly well in practice.

Once you have selected which (old) free variable is to become a (new) basic variable, you have to choose a basic variable to take its place. **Usually there is only one right choice!** Each (old) basic variable has an associated cost, and you **must** choose the basic variable with the **least cost**. Occasionally there are ties. If you use Bland’s rule, then it also asks you to choose the left-most basic variable with the least cost to be the new free variable (a tie-breaking rule). If you use the greedy rule, you can run into infinite loops (ties can ruin your day). This is an informal summer class, so we will not need jackets or ties.

**Cost:** Once you have chosen a free variable to make basic, you have chosen a **pivot column**. For each row, if that row has a **positive number** in that pivot column, then it has a cost: take the RHS for that row, and divide it by the value from that row of the pivot column. All costs should be positive (if not, you made a mistake earlier, see Ch 4.3 for how to recover, or just start over). Choose the row with the **least cost**. If there are ties, then choose randomly (greedy rule) or choose the row whose variable is left-most (Bland’s rule). The row you choose is the **pivot row**, and the pivot row and pivot column together tell you the **pivot**. The pivot row should have a 1 in exactly one of the columns corresponding to old basic variables. That column’s variable is the old basic variable that will become a new free variable.

The old free variable used to be unimportant (it was going to be set to 0). The old basic variable used to be important (it was allowed to be nonzero). Now their roles are reversed.