Chapter 4: Practice Exam

MA162-020
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1. Setup the problem but do not solve. Write the variables, the constraints, and the objective.

Boise Lumber has decided to enter the lucrative pre-fabricated housing business. Initially it plans to offer three models: standard, deluxe, and luxury. Each house is prefabricated and partially assembled in the factory, and the final assembly is completed on site. The standard model costs \$6000 in materials, requires 240 hours of factory labor, 180 hours of on-site labor, and sells for a \$3400 profit. The deluxe model requires \$8000 in materials, 220 hours of factory labor, 210 hours of on-site labor, and sells for a \$4000 profit. The luxury model requires \$10000 in materials, 200 hours of factory labor, and 300 hours of on-site labor. Boise Lumber has allocated \$8.2 million in materials, 218 thousand factory hours, and 237 thousand on-site hours for the first year of production. How many houses of each type should be produced in order to maximize the profit while staying within the budget?

2. Write down the (standard, primal) tableau corresponding to the problem:

Maximize $P = 3x+4y+5z$ subject to $6x+7y+8z \leq 100$, $11x+10y+9z \leq 101$, $13x+14y+12z \leq 102$, $15x+17y+16z \leq 103$, $x, y, z \geq 0$.

3. Apply 1 step of the simplex algorithm. Circle your pivot, write our your row ops, and write down the next tableau. Explain why that next tableau is final or not final.

x	y	z	u	v	w	P	RHS
1	1	−3	1	0	0	0	1
2	1	0	0	1	0	0	3
−4	3	0	0	0	1	0	2
−3	2	2	0	0	0	1	0

4. Read the answer from the following finished tableau (based on #1). Give the location of the maximum, the maximum itself, and the resulting surpluses. Feel free to use #1 to give a plain English version of the answer for the owner of Boise Lumber.

X	Y	Z	M	F	S	P	RHS
0	1	0	1	0	−1/3	0	300
0	0	1	−3/10	−1/20	1/6	0	400
1	0	0	−2/3	1/12	1/6	0	300
0	0	0	7/30	1/30	1/15	1	4220

5. Write down the dual tableau of the problem: Minimize $C = 3x+2y+4z$ subject to $5x+6y+7z \geq 500$, $8x+9z \geq 720$, $x, y, z \geq 0$. After pivoting for a few hours, you get the final dual tableau. Write down the solution to the original (primal) minimization problem, including the location of the minimum, the minimum itself, and the surpluses.

u	v	x	y	z	C	RHS
0	1	1/8	−5/48	0	0	1/6
1	0	0	1/6	0	0	1/3
0	0	−9/8	−11/48	1	0	1/6
0	0	90	25/3	0	1	860/3

1. The free variables are the number of houses of each model to produce, say $X = \#of Standard models to produc$
$Y =$ Deluxe, $Z =$ Luxury. The problem then is to maximize $P = 3.4X + 4Y + 5Z$ subject to $6X + 8Y + 10Z \leq 8200$ (materials), $240X + 220Y + 200Z \leq 218000$ (factory hours), $180X + 210Y + 300Z \leq 237000$ (on-site hours), and $X, Y, Z \geq 0$ (sanity).

You can also name the surpluses: $M =$ thousands of dollars of left-over materials, $F =$ hours of unused factory labor, $S =$ hours of unused on-site labor. The problem is then: maximize $P = 3.4X + 4Y + 5Z$ subject to $M = 8200 - 6X - 8Y - 10Z$, $F = 218000 - 240X - 220Y - 200Z$, $S = 237000 - 180X - 210Y - 300Z$, and $X, Y, Z, M, F, S \geq 0$.

2. Maximize $P = 3x+4y+5z$ subject to $6x+7y+8z \leq 100$, $11x+10y+9z \leq 101$, $13x+14y+12z \leq 102$, $15x + 17y + 16z \leq 103$, $x, y, z \geq 0$.

x	y	z	s	t	u	v	P	RHS
6	7	8	1	0	0	0	0	100
11	10	9	0	1	0	0	0	101
13	14	12	0	0	1	0	0	102
15	17	16	0	0	0	1	0	103
−3	−4	−5	0	0	0	0	1	0

3. The only reasonable pivot column is the first, since that is the only column with a negative in the bottom row. The only reasonable pivot row is the first, since it has the smallest freedom (1/1 versus 3/2 versus negative). Pivot on the top-left entry, making x basic and u free.

x	y	z	u	v	w	P	RHS
1	1	−3	1	0	0	0	1
0	−1	6	−2	1	0	0	1
0	7	−12	4	0	1	0	6
0	5	−7	3	0	0	1	3

The resulting tableau is not final, since increasing z will still increase profit, so one would need to do more steps to get a final tableau, the first step presumably being to pivot the third column and second row.

4. We see that $P = 4220 - (7/30)M - (1/30)F - (1/15)S$, and so we clearly decide $M = 0$, $F = 0$, $S = 0$. We can then ignore those columns and read the resulting (mixed up) RREF to get $Y = 300$, $Z = 400$, $X = 300$, $P = 4220$. In plainer language, we will use all our resources to make 300 standard, 300 deluxe, and 400 luxury models, for a total profit of 4.22 million dollars.

5. $x = 90$, $y = 25/3$, $z = 0$, $C = 860/3$. The surpluses are all 0, either by direct calculation (as on the quiz), or by reading the bottom row.