Name: _____

Quiz 5.2: Annuities

Write the formula, the values of the known variables, and box the value of the sought-after variable.

1. If you invest \$300 per month at 5.90% compounded monthly, how much will your investment be worth in 25 years?

2. A company contributes \$180 per month into a retirement fund paying 4.30% compounded monthly and employees are permitted to invest up to \$2,400 per year into another retirement fund which pays 4.30% compounded annually. How large can the combined retirement fund be worth in 26 years?

3. If you invest \$23,000 at 8% compounded quarterly, after how many years, rounded to .01, will your investment be worth \$72,533.36?

4. A large corporation lost a warehouse in a fire. They received a check for \$200,000 from the insurance company, and the land was appraised at \$50,000. They are considering building on office complex. It would cost \$300,000 and take a year to build, but its expected value at the end of the year is \$400,000. Of course, the capital market offers trading in securities with similar risk profiles that have an expected rate of return of 7%. Which leads to the larger increase in net present value: building the office complex or investing the money in the securities with similar risk profiles?

Examples 5.2: Annuities

One rarely invests a fixed amount of money for a fixed period of time. Compound interest is just a building block for more complicated financial instruments. The next simplest instrument is a fixed cash flow called an **annuity**. Every period a fixed amount of money is invested into an account earning a fixed interest rate (that is, we assume a "fixed payment schedule" and a "flat term structure" of interest). We also assume that the payments are made at the end of the conversion period (so do not immediately earn interest). One is interested in (a) how much the account will be worth in the future, and (b) effectively how much that future account is worth now.

The formula for the **future value of the annuity** is:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

- R is the recurring payment made at the end of each period
- i is the interest rate on the account, per period
- n is the number of periods

To find the present value of future money can be very easy: how much do you need to invest now in order to have that much money in the future? This is called **depreciating** the money. If the prevailing interest rate (the rate of return of securities with a similar risk profile in the capital markets) is i, and the "future" is n conversion periods away, then A future dollars are worth

$$P = A/(1+i)^n$$

present dollars now.

Some people like to plug this in directly to the annuity formula, but the formula is very easy to misremember and involves negative exponents, so be careful:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

It is also a good idea to remember the following vocabulary:

- 1 times per year: **Annually**
- 2 times per year: **Semi-annually**
- 4 times per year: **Quarterly**
- 12 times per year: **Monthly**
- 26 times per year: **Biweekly**
- 52 times per year: **Weekly**
- 360 times per year: **Daily**

It is also a good idea to learn to convert these time units quickly. For instance, 3 months is 1 quarter, but is also 0.25 years. For another example 10 months is $\frac{5}{6}$ of a year, and 17 months is $\frac{17}{12}$ of a year, also known as 1 year and 5 (more) months.