# Practice Exam: Chapter 5



For each problem, be sure to show the formula used, and indicate the values of the variables. Like $A = P(1 + i)^n$, $P = \$10$, $i = 4\%/12$, $n = 25(12) = 300$.

1. Gary knows that he will need \$18,000 after about 6 years from now to buy new printers for his shop. He also notices that his current savings are not making much money and he decides to invest them in a growth fund to have enough money available after 6 years. (a) How much would he have to invest now at 8% annual rate compounded annually to have that sum available? (b) Suppose he finds a better investment opportunity which offers bi-weekly compounding at the same annual rate. Assuming 26 nominal bi-weeks in the year, how much would he need to invest in the new scheme? Be sure to show the formulas used.

2. You are about to finance the purchase of a new house with a 20 year loan of 110 thousand dollars at 3.7% APR compounded monthly. (a) You are supposed to pay monthly. What is your monthly payment and what is your total of the payments over the period of 20 years? (b) Suppose that another lender had offered the same loan with the same interest rate but for 30 years instead of 20 years. What will be the new monthly payment and the total of the payments over the period of 30 years?

3. John got a new job and is eager to replace his old car with a new one. But he does not want to take the responsibility of a new car loan. Therefore, he starts putting money into a savings account so he can buy a car after 5 years. John figures that he would need to have \$14,000 saved, and he wishes to make monthly payments into an account paying 4% interest compounded monthly. Help him figure out the following. (a) How much should John's monthly payment be? (b) At the end of 5 years, John decides to continue with his old car for another year yet continues making the same payments, so he can get a better car. How much additional money will accumulate into the account?

4. In this problem, assume a 360 day year. "Payday!" will loan you 55% of your paycheck of \$700 for 10 days. After 10 days, you pay back the loan plus an interest of \$24. (a) Calculate the annual simple interest rate for this service. (b) Across the street, "Cash Now!" will loan you 5% more of your paycheck but charge you an interest of \$26. Is this a lower rate?

5. You've got \$23 and have found two banks that offer 8% interest. Lenny's Loans and Loot Conversion offers 8% simple interest, and Barry's Bank and Bingo Quarters offers 8% compound interest. (a) How long would it take your investment to grow to \$100 at Lenny's LLC? (b) How long would it take your investment to grow to \$100 at Barry's BBQ?

1. This is a compound interest problem because of the phrase "invest now" and "compounded annually". It is not a simple interest problem, since simple interest is not mentioned, and it is not an annuity problem since there is no mention of investing money periodically.

| Part (a) | Part (b) |
|---|---|
| $A = \$18,000$ | $A = \$18,000$ |
| $n = 6$ years | $n = (6 \text{ years})(26 \text{ biweeks per year})$ |
| | $= 156$ biweeks |
| $i = 8\%$ per year | $i = (8\% \text{ per year})(1/26 \text{ year per biweek})$ |
| | $= (8/26)\%$ per biweek |
| $P = \ ?$ | $P = \ ?$ |
| $A = P(1+i)^n$ | $A = P(1+i)^n$ |
| $P = A/(1+i)^n$ | $P = A/(1+i)^n$ |
| $P = \$18000/(1+0.08/1)^6$ | $P = \$18000/(1+0.08/26)^{(26)(6)}$ |
| $P = \$11343.05$ | $P = \$11146.31$ |

2. This is an annuity problem, since "you . . . pay monthly". In fact it is exactly the kind of annuity problem worked in the examples of chapter 5.3. In particular, we want to use the present value of the cash flow, not the future/accumulated value.

| Part (a) | Part (b) |
|---|---|
| $P = \$110000$ | $P = \$110000$ |
| $n = (20 \text{ years})(12 \text{ months per year})$ | $n = (30 \text{ years})(12 \text{ months per year})$ |
| $= 240$ months | $= 360$ months |
| $i = (3.7\% \text{ per year})(1/12 \text{ year per month})$ | $i = (3.7\% \text{ per year})(1/12 \text{ year per month})$ |
| $= (3.7/12)\%$ per month | $= (3.7/12)\%$ per month |
| $R = \ ?$ | $R = \ ?$ |
| $T = \ ?$ | $T = \ ?$ |
| $P = R(1-(1+i)^{-n})/(i)$ | $P = R(1-(1+i)^{-n})/(i)$ |
| $T = Rn$ | $T = Rn$ |
| $R = P/((1-(1+i)^{-n})/(i))$ | $R = P/((1-(1+i)^{-n})/(i))$ |
| $R = 110000/((1-(1+0.037/12)^{-240})/(0.037/12))$ | $R = 110000/((1-(1+0.037/12)^{-360})/(0.037/12))$ |
| $R = \$649.32$ | $R = \$506.31$ |
| $T = (\$649.32)(240)$ | $T = (\$506.31)(360)$ |
| $T = \$155836.42$ | $T = \$182272.06$ |

3. This is an annuity problem because "he ... make[s] monthly payments". Since he wants the money in the future, this is the accumulated / summed / future value of the annuity, as in chapter 5.2.

| Part (a) | Part (b) |
|---|---|
| $S = \$14000$ | $S = \ ?$ |
| $n =$ (5 years)(12 months per year) | $n =$ (6 years)(12 months per year) |
| $= 60$ months | $= 72$ months |
| $i =$ (4% per year)(1/12 year per month) | $i =$ (4% per year)(1/12 year per month) |
| $= (1/3)\%$ per month | $= (1/3)\%$ per month |
| $R = \ ?$ | $R =$ (whatever we get in part (a)) |
| $S = R((1+i)^n - 1)/(i)$ | $S = R((1+i)^n - 1)/(i)$ |
| $R = S/(((1+i)^n - 1)/(i))$ | |
| $R = \$14000/(((1+0.04/12)^{60} - 1)/(0.04/12))$ | |
| $R = \$211.16$ per month | $S = \$211.16((1+0.04/12)^{72} - 1)/(0.04/12)$ |
| | $S = \$17150.95,$ which is |
| | $\$17150.95 - \$14000 = \$3150.95$ additional |

4. This is simple interest because it says "annual simple interest rate".

| Part (a) | Part (b) |
|---|---|
| $P = (\$700)(55\%)$ | $P = (\$700)(60\%)$ |
| $= \$385$ | $= \$420$ |
| $t =$ (10 days)(1/360 years per day) | $t =$ (10 days)(1/360 years per day) |
| $= (1/36)$ year | $= (1/36)$ year |
| $I = \$24$ | $I = \$26$ |
| $I = Prt$ | $I = Prt$ |
| $r = I/(Pt)$ | $r = I/(Pt)$ |
| $r = (\$24/\$385)/(1/36)$ | $r = (\$26/\$420)/(1/36)$ |
| $r = 224.42\%$ | $r = 222.86\%$ |

so "Cash Now!" is a better deal, or at least, slightly less of a rip-off. Remember the calculator says 2.2416, but expressed as a percentage, that is 224.16%.

Part (a) is simple interest, and part (b) is compound interest. Neither is an annuity. The problem has a typo, part (b) should specify the conversion period. We'll assume annual conversion.

| Part (a) | Part (b) |
|---|---|
| $P = \$23$ | $P = \$23$ |
| $r = 8\%$ | $i = 8\%$ |
| $t = \ ?$ | $n = \ ?$ |
| $I = \$100 - \$23 = \$77$ | $A = \$100$ |
| $I = Prt$ | $A = P(1+i)^n$ |
| $t = I/(Pr)$ | $n = \log(A/p)/\log(1+i)$ |
| $t = (\$77/\$23)/(0.08)$ | $n = \log(\$100/\$23)/\log(1+0.08)$ |
| $t = 41.85$ years | $n = 19.10$years |

Barry's BBQ is the faster way to turn \$23 into \$100. However, a part time job or bake sale is probably quite a bit faster than the 19 some odd years required for the interest to accumulate.