Name: ______________________________

MA162-020
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Quiz 6.2: Counting by Inclusion-Exclusion

A survey for the student center food services reports that: 130 students ate breakfast, 180 ate lunch, and 275 ate dinner. Realizing they needed more details, they asked the same group of students some more questions and reported that: 68 students ate both breakfast and lunch, 112 ate both breakfast and dinner, and 90 ate both lunch and dinner. Realizing they needed yet more detail, they asked the same group of students one more question and reported that: 58 students ate all three meals.

1. How many of the surveyed students ate at least one meal?

2. How many of the surveyed students ate exactly one meal?

3. How many of the surveyed students ate only dinner?

4. How many of the surveyed students ate exactly two meals?

Examples 6.2

We use the symbols $n(A)$ to mean the number of elements of the set A. Our first (fancy) counting formula is known as inclusion-exclusion:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

For instance if $A = \{T, W, R\}$ and $B = \{M, T, W\}$, then $A \cup B = \{M, T, W, R\}$ and $A \cap B = \{T, W\}$ so to count the days we have covered we have: 3 days from Adam, plus 3 days from Bianca, but that double counts $A \cap B = \{T, W\}$, so we subtract 2. In other words we have $3 + 3 - 2 = 4$ days covered.

Inclusion-exclusion works with more than two sets. We'll also use the three set version:

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

This works for the same reason: we add up A, B, and C, but that means we double counted everything in the overlaps, so we subtract the overlaps $A \cap B$, $A \cap C$, and $B \cap C$, but that leaves the triple overlap $A \cap B \cap C$ in a weird state: we triple counted it the first time through, and then we triple-uncounted it during the subtraction, so we end up having not counted it at all, so we add in one copy of $A \cap B \cap C$.

I suggest working exercises 1-36 in 6.2. Exercises 29-36 are practice versions of an exam question that many have found difficult to do without practice.

Smaller quiz:

1. In a survey of 200 households, it was found that 120 owned only desktop computers, 10 owned only laptop computers, and 40 owned neither. How many owned both desktop and laptop computers?

2. A certain drug-test is "99% accurate!!" as in, it reports 99% of users as users, 1% of users as non-users, 99% of non-users as non-users, and 1% of non-users as users. If this drug-test reports 10 people out of 510 are users, then:

(a) How many people out of 510 are users?

(b) On average, how many people out of the 10 reported are users?

(c) What percentage accuracy would you say the test really has for 10 people out of 510?