

Chapter 6: Practice Exam

MA162-020
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1. Suppose that P , Q and R are sets with 40, 41, and 33 elements respectively. Calculate the indicated quantities. Display correct formulas or appropriate Venn diagrams. (i) If $P \cap Q$ has 17 elements, then $P \cup Q$ has how many elements? (ii) If it is further known that $P \cap R$ has 15 elements, then $P \cup R$ has how many elements? (iii) If, in addition, $Q - R$ has 26 elements, then $Q \cap R$ has how many elements? (iv) Finally, if we are given that the intersection of all three sets P , Q , and R has 9 elements, then the union of all three sets has how many elements?

2. A survey of 100 College students revealed the following:

- 39 students use the Radio for their news.
- 42 students use TV for their news.
- 42 students use the Internet for their news.
- 14 students use both Radio and TV for their news.
- 14 students use Radio and Internet for their news.
- 15 students use TV and Internet for their news.
- 11 students use all three news sources.

Based on the above information, answer the following questions. You must show your calculations to receive credit. (a) How many students surveyed use none of the three news sources? (b) How many students surveyed use at least two of the three news sources?

3. Mona does not like to wear the same outfit twice, so she shops at *Swatches*, the hip new interchangeable fashion store for today's fashion elite. She has 5 different shirts, 6 different pants, and 8 different pairs of shoes. (a) How many successive days can she wear an outfit without repetition? (b) Mona goes out and buys two new shirts but unfortunately one of her shoes breaks down. Now, how many days can she go without repeating an outfit?

4. There are 24 members in an executive committee. A subcommittee needs to be appointed for a special project. The subcommittee shall have a chairman, a public relations officer, a secretary and an accountant. Answer the following questions. Be sure to show your reasoning. Just numerical answers shall earn no credit. (a) How many different subcommittees can be formed from the executive committee? (b) Suppose that the regular secretary of the executive committee has been assigned as the secretary of the sub committee. How many different subcommittees can now be formed?

5. If you roll two (distinguishable, six-sided, fair) dice, how many possible outcomes are there? How many of those are "doubles", that is, where both dice have the same number? How many of those are "odd", that is, where the sum of the two dice is an odd number? How many of those are an "eight", that is, where the sum of the two dice is 8?

1. (i) $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q) = 40 + 41 - 17 = 64$. (ii) $n(P \cup R) = n(P) + n(R) - n(P \cap R) = 40 + 33 - 15 = 58$. (iii) $n(Q) = n(Q - R) + n(Q \cap R)$, since everything in Q is either in R (and so in $Q \cap R$), or not (and so in $Q - R$). So $n(Q \cap R) = n(Q) - n(Q - R) = 41 - 26 = 15$. (iv) $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(P \cap R) - n(Q \cap R) + n(P \cap Q \cap R) = 40 + 41 + 33 - 17 - 15 - 15 + 9 = 76$.

2. (a) $n(\text{None}) = 100 - n(R \cup T \cup I)$. $n(R \cup T \cup I) = n(R) + n(T) + n(I) - n(R \cap T) - n(R \cap I) - n(T \cap I) + n(R \cap T \cap I) = 39 + 42 + 42 - 14 - 14 - 15 + 11 = 91$ use at least one, so 9 use none. (b) If we add up $14 + 14 + 15$, then we have counted all and only the people who use at least two of the three news sources, but we have counted the ones who use all three 3 times instead of just once, so we need to subtract 11 twice: $14 + 14 + 15 - 11 - 11 = 21$.

3. (a) $(5)(6)(8) = 240$. (b) $(5 + 2)(6)(8 - 1) = 294$.

4. (a) $(24)(23)(22)(21) = 255024$. (b) $(23)(22)(21) = 10626$.

5. (a) $(6)(6) = 36$. (b) 6. (c) Half, 18. (d) $n(\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}) = 5$.