Name: ______________________________

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Quiz 7.4: Computing some compound probabilities

1. A coin is tossed five times. What are the following probabilities:

(a) Five heads

(b) Exactly one heads

(c) At least one heads

(d) More than one heads

2. Two cards are drawn from a deck. What are the following probabilities:

(a) A pair is drawn

(b) A pair is not drawn

(c) Two red cards are drawn

(d) Two cards of the same suit are drawn

3. Families with three (non-twins, non-triplets, standardly gendered) children are selected at random. What are the following probabilities of children, assuming a boy and a girl are equally likely:

(a) Two boys and one girl

(b) At least one girl

(c) No girls

(d) The two oldest are girls

Examples 7.4: Compound probabilities

We can use counting techniques to compute probabilities in uniform sample spaces that are too large to write out completely.

Example: For extra security, a high school searches 15 random lockers each school day. If there are 540 students with lockers, what is the probability that a particular student's locker gets searched at least once in a 40 day period?

Each day there are $(540)(539)\cdots(526)/((15)(14)\cdots(1))$ choices of 15 lockers out of 540 to search. If we want to avoid getting searched, then that means there are only 539 "good" lockers for them to search, so $(539)(538)\cdots(525)/((15)(14)\cdots(1))$ "good" choices of 15 lockers to search. The probability of not being searched on one day is thus:

$$\frac{n(\text{"good choices"})}{n(\text{"all choices"})} = \frac{\frac{539}{15}\cdot\frac{538}{14}\cdots\frac{525}{1}}{\frac{540}{15}\cdot\frac{539}{14}\cdots\frac{526}{1}} = \frac{525}{540} = 1 - \frac{15}{540}$$

Now the probability of not being searched two days in a row is $(1-\frac{15}{540})^2$, assuming they choose randomly and independently each day. For three days, you just need tog et lucky one more time, so $(1-\frac{15}{540})^3$ overall. For 40 days, to not get searched only has a $(1-\frac{15}{540})^{40}$ chance, so the probability to get searched at least once is:

$$1-(1-\tfrac{15}{540})^{40} \approx 67.59\%$$

Example: Bob and Sue are in a group of five people who will be assigned places around a table randomly. What are the odds that Bob and Sue sit next to each other?

There are $(5)(4)(3)(2)(1)/(5)$ seating arrangements. Since we can label the seating arrangement starting with Bob and then only have $(4)(3)(2)(1)$ arrangements (still 24), we can count the number of ways Sue sits on his left as $(1)(3)(2)(1)$, and the number of ways she sits on his right as $(3)(2)(1)(1)$, for a total of 12. So there are 12 "good" ways out of 24 possible ways, so 50% probability.

We could also say "let Bob sit down first. There are four seats left, two are next to him and two are not, so Sue has a $2/4 = 50\%$ chance to sit next to him."